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THE MATHEMATICS TEACHER

Volume XXVIII Number 8

Edited by William David Reeve

Teaching for Transfer

By HAROLD P. FAWCETT
University High School, Columbus, Ohio

OUT OF the confusion of viewpoints relative to the transfer of training there seems to be one general agreement which is clear and well defined: transfer is not automatic. If as teachers of mathematics, we believe that there are values in our subject which are able to change the behavior of a pupil in such a way as to make him a more useful citizen and better equipped to solve the problems of life, we have no right to expect that this desirable result may be accomplished by simply hoping that it will follow inevitably from any kind of teaching.

Since it is generally agreed that if we would have transfer, we must teach for it, we are at once faced with the problem of determining what there is in mathematics which is worthy of transfer. If transfer is a function of teaching, then certainly every teacher of mathematics should have clearly defined, for himself at least, those values which he believes will serve to illumine and clarify the various phases of life. What are those values? In attempting to answer this question we naturally think first of the practical side of our subject. Mathematics was born in an effort to solve the problems which confronted a developing civilization and now in the twentieth century it serves the same purpose. There is still value in

knowing how to use a formula, interpret a graph, or solve an equation.

Within reasonable limits these practical values of mathematics have been pretty well determined and to teach the necessary skills in such a way that they can be used when needed is a problem about which much has been written and it is not my purpose to deal with it here. I prefer rather to consider another kind of value which in my opinion mathematics is peculiarly fitted to cultivate. Mathematics has been called a "way of thinking" and Professor Keyser has said that "wherever inferences are drawn logically, mathematical thinking occurs." Now as teachers of mathematics are we not interested in improving the quality and extending the scope of this thinking? Is there any other area in the entire curriculum of the secondary school that is so peculiarly qualified to give pupils training in "drawing inferences logically" as that of mathematics. Here is a fertile field with unlimited possibilities of transfer for there is no phase of human activity where logical inference does not play a vital part, and yet I wonder if the boys and girls who pass through our classrooms and depart with our mathematical blessing are any better equipped because of that experience to draw correct inferences from the kind of data they meet in the various avenues of life. Of course we teach demonstrative geometry and we justify it by saying that the real purpose is to train the pupil in deductive thinking and to acquaint him with the nature of proof. These are indeed worthy purposes but in view of the fact that any worth-while transfer results only when there is conscious teaching for it, does it not seem desirable to extend the study of proof beyond the narrow confines of geometry and examine some of the countless propositions that are met in other areas of life?

To illustrate this I should like to describe briefly something of what is being done in the University High School at Ohio State University. If the real purpose of teaching demonstrative geometry is to give the pupil an understanding of the nature of proof, the emphasis should not be placed on the conclusions reached, but rather on the kind of thinking used in reaching these conclusions. With this in mind we began our study with a consideration of the importance of definitions and agreements in many of the activities of life. Illustrations were drawn from different sorts of activities, but perhaps those which were most impressive related to games. In baseball, for example, the pupils themselves eagerly pointed out

that a pitched ball in order to be a strike had to meet certain requirements which satisfied the definition of a strike. The same is true of a ball, a foul, a home run and other such concepts all of which are defined in the domain of baseball. Football was also considered and the necessity of defining such fundamentals as touchdown, field goal, safety and touchback was discussed. We looked upon the rules of the game as agreements which both officials and players must respect; using the definitions and rules which are now recognized, we have the game as it is played this year, 1935. Last year some aspects of the game were different. The pupils quickly volunteered the reason for this, pointing out that some of the rules were different. A change in one or more of the agreements then produces a difference in the game. A difference in the game also occurs if there is a change in one or more of the definitions. Much emphasis was given to these considerations and the pupils learned to see something of the importance of definitions and agreements in this sort of familiar activity. These definitions and agreements imply certain conclusions relative to the activities of the players and it was carefully pointed out that any change in either a definition or an agreement was likely to change the conclusion.

Through such considerations we were led to a realization that any theory in any area is based on a limited number of terms, defined and undefined, and on certain agreements, which may properly be called assumptions. We then decided to build a theory of our own in connection with some interesting area and at this point the pupils were very definitely guided into building a theory of space where the concepts are impersonal and do not have a tendency to stir the emotions. We began by recognizing the fact that a small number of undefined or primitive terms were necessary and after listing many suggestions as to what these terms might be, we finally narrowed them down to point, line, plane and angle. All other terms were defined by means of these primitive terms, but no term was introduced until it was needed. Then in our effort to build a theory we realized that a set of agreements or assumptions was necessary, but these also were introduced only as needed. The pupils would study a diagram and infer that it possessed certain properties. They thus set up a theory concerning this particular diagram and in attempting to justify their conclusions the need for agreements and assumptions was felt. These

assumptions were then established. In this manner we have reached a number of conclusions relative to the properties of geometric figures both of two and three dimensions.

Now the pupils do not regard these conclusions as absolutely true. They very definitely recognize that they depend on the definitions and assumptions from which they were deduced and they also recognize that they are true only to the degree that these definitions and assumptions are true. A change in any one of the definitions and assumptions would probably change one or more of the conclusions. There is no such thing as absolute truth. This was made real to them through a careful study of the theorem relating to the sum of the interior angles of a triangle. We thoroughly analyzed the so-called proof of this theorem and listed the definitions and assumptions used in establishing the conclusion. Among these is found the famous assumption that "through a given point not on a given line one and only one line can be drawn parallel to a given line." We do not have to wonder what would happen if this assumption were changed for we know that it is not accepted in either the geometry of Lobachevsky or in that of Riemann. In the first of these the sum of the interior angles of a triangle is less than 180°, while in the other the sum of these angles is greater than 180°. This did not surprise the pupils as much as one might expect, but it did interest them greatly and quite definitely deepened their acquaintance with the "nature of proof." They went to books on the history of mathematics, learned how non-Euclidean geometry developed and became acquainted with Lobachevsky, Bolyai, Gauss and Riemann.

We then proceeded to apply this same kind of thinking to the Declaration of Independence. The pupils analyzed this on the following basis:

1. What is the conclusion reached by the men who signed this document?
2. What are the assumptions on which the conclusion is based?
3. What are some of the arguments advanced to establish this conclusion?

These analyses were excellent. In discussing them many questions such as the following were raised by the pupils:

a. Would there have been a Civil War if the assumption had been that all white men are created equal?

b. How is "equal" defined?
c. Is the assumption that "all men are created equal" correct?
d. What difference would there have been in the government if the signers had assumed that only the rich were "endowed by their Creator with certain inalienable rights"?
e. Would there have been a Civil War if the signers had assumed that black men had no rights?

This by no means exhausts the questions asked, but it gives a fair sample of how the children were thinking.

The kind of advertising to which students are daily exposed is another fertile field in connection with the idea of proof. Behind every advertisement is a multitude of assumptions. The pupils have become greatly interested in listing the assumptions made by the person who buys the advertised article through the influence of the advertisement. On the whole it has had a decided influence in their ability to detect these assumptions and in many cases the pupils have stated that they are becoming more resistant to advertising and are looking for proof of the facts stated. Many of them have voluntarily submitted their analyses of advertisements which they have found in their reading with a further statement concerning the assumptions which they accept and those which they reject. This illustrates how their thinking about matters involving proof has been changed.

To emphasize the importance of definition we select statements from current literature which have little meaning until certain significant words or phrases in these statements are defined. These are given to the pupils and they select the words or phrases on which the meaning of the statement depends. For example, when a candidate for a political office says that the party "pledges an adequate program of state taxation to be used for educational purposes" surely any thoughtful person would like to know more definitely how he defines the word "adequate." Through this kind of exercise we have made very definite progress in having the pupils understand that words may mean one thing to one person and another thing to another and that much misunderstanding between people may be traced to a conflict of definitions. While we have many illustrations of the way this is affecting the thinking of the pupils perhaps the most interesting is the reaction which comes from the social science area where the pupils are insisting

on the use of clearly defined conditions and are raising questions as to what are the assumptions behind some of the social theories discussed.

The field of science is also fertile with possibilities as far as assumptions are concerned. In cooperation with the science teacher we have emphasized the importance of assumptions in this field and have given the pupils considerable practice in selecting the assumptions on which certain simple principles of science depend.

Since we have accepted as a fundamental assumption Professor Keyser's statement that "wherever inferences are drawn logically, mathematical thinking occurs," we have a key which opens the mathematics classroom to all phases of human activity. But you may begin to wonder how much of the usual subject matter of geometry the pupils are covering. In answer to this I would say that we are not at all interested in how many geometric theorems the pupils study. If the primary purpose in teaching demonstrative geometry is to acquaint pupils with the "nature of proof," no one can say how many theorems it is necessary to prove in order that this result may be achieved. I did however plan to cover the twenty essential theorems and the ten essential constructions listed in Dr. Christofferson's book "Geometry Professionalized for Teachers." However, I find that it is practically impossible to limit what the pupils do in this way. No formal text is being used. The pupils are writing their own text as this work develops in the classroom and one of the most stimulating things that has occurred is the fact that they, themselves, have discovered many theorems which we have included in the development. Lists of original exercises have also been provided for those who wish to consider them and, although unassigned, large numbers of these have been worked out by a majority of the pupils. From my point of view, however, the important thing is not how many theorems are covered or how many original exercises have been proved. I am more concerned with the pupil's understanding of what it really means to prove something and demonstrative geometry offers a perfect illustration of such proof. In no other kind of activity is it possible to show so vividly how even the smallest change in a definition or an assumption affects the conclusion. This, it seems to me, is one of the most important elements in the nature of proof and to illustrate the extent to which the pupils are getting some conception of this let me quote from the paper of a thirteen-year-

old boy who says he is trying to answer the question "What is a proof?"

When we have certain statements which we accept and find that those statements imply other statements, we have a proof. So, by my definition of proof, all proofs can be written in the 'If . . . then' form and no conclusion is true unless all the 'ifs,' usually assumptions and definitions, are also true.

There are two kinds of proofs, the inductive and the deductive. The inductive proof is based on the assumption that: if a fact is tested a comparatively small number of time, k, and is found to be true for those k times, then the fact is true. Thus the formula n^2-n+41 should always give you a prime number for that is what happens when n equals from 1 to 25. But n^2-n+41 does not always give you a prime number since when $n=41$ we have $1681-41+41=1681=41^2$. Therefore the assumption about testing a fact k times is false.

All deductive proofs are ultimately based on inductive proofs. Assumptions are necessary in deductive proofs and all assumptions are proved inductively if proved at all. And since a fact is no truer than its component parts a deductive proof cannot be considered to be true. However Euclid made such obvious assumptions, there is only one that I am doubtful of, that in general people have accepted them for 2000 years. So the theorems which Euclid proved have been considered to be absolutely true. A danger in a deductive proof is that there may be some hidden assumption which you will believe. But in a geometric proof it is much easier to detect the hidden assumption than in an advertisement. It is much easier to convince you that you should buy this medicine for colds even if it has five harmful drugs in it than to convince you that 1 equals 2.

In proving that the sum of the exterior angles of an n-gon equals 360°, I noticed one assumption that the conclusion rested on. That assumption was that: one and only one straight line can be drawn through a given point parallel to a given line. And that assumption seems much less certain than the fact of gravitation which has been tested millions of times. So I am much more certain that if I let go of my pen two feet above the ground it will drop, than that the sum of the exterior angles of an n-gon equals 360°.

A number of other students have voluntarily written on this topic and all of their papers reveal their increasing grasp of what it really means to prove something. One girl for example after mentioning both inductive and deductive proof says that "in inductive proof, if under the same circumstances a certain thing has happened every time, then you feel that it will happen the next time these circumstances occur, although you cannot absolutely prove it." She then gives an excellent illustration of this and proceeds as follows:

In deductive proof, by carefully worked out steps you lead up to an inevitable conclusion. This conclusion is accepted by all people in so far as the assumptions and definitions behind it are accepted by the persons concerned.

Take an example from the field of geometry. In the proposition, the sum of the

interior angles of a triangle is one hundred and eighty degrees, the conclusion depends upon the assumption, among others, that one and only one line can be drawn through a given point parallel to a given line. If anyone should dispute this assumption or change it, they can prove that the sum of the interior angles of a triangle is not one hundred and eighty degrees. Other assumptions may be selected by which they can prove that the sum of the interior angles of a triangle is either greater than one hundred and eighty degrees or less than one hundred and eighty degrees.

Therefore, we see that deductive proof yields a conclusion which is true only within the limits of the assumptions and definitions upon which it rests.

One of the most valuable developments in scientific thought during the nineteenth century is the realization that all the conclusions of mathematics, considered for so long to be absolutely true, depend in the final analysis on a series of assumptions. This indeed is true of the conclusions reached in any area of thought and to make pupils conscious of this tremendous fact is to equip them with a powerful method of thinking about the problems of modern life. It seems to me that demonstrative geometry is peculiarly qualified to cultivate this type of thinking, but it will not serve this purpose as long as the teacher is required to cover a given number of theorems.

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A Proposed National Committee on Arithmetic*

By R. L. MORTON
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THERE ARE many respects in which books dealing with the teaching of arithmetic disagree. Textbooks designed for the use of children also disagree as to content, the arrangement of the material selected, and the suggested methods of procedure. Some of the disagreements are implied in the following questions which represent more or less random selections from the arithmetic of the elementary school.

1. What is the place of rote counting in teaching number?
2. In what grade and at what mental age should formal instruction in arithmetic be begun?
3. What should be the nature and the extent of so-called incidental number teaching prior to the beginning of formal instruction?
4. Should the order in which the basic combinations of addition are presented be one in which there is a sequence of those having identical sums, or one in which one of the addends of those presented consecutively is common while the others differ but one, or any one of a number of variations of these plans; or, should the order be one in which all such plans are carefully avoided in an effort to develop habits of arriving at sums without the use of counting or other circuitous means. What procedure will secure the best balance between an understanding of number and number relationships on the one hand and the development of adequate addition habits on the other, assuming that there may be a conflict between these two points of view?
5. Which of the methods of subtraction should be used in introductory work in this process? If one method is adopted for introductory work, to what extent should other methods be used in solving problems involving the various ideas of subtraction?

* Read at the Annual Meeting of the National Council of Teachers of Mathematics at Atlantic City, N. J. on Saturday morning, February 23, 1935.

6. In presenting the basic combinations of multiplication, to what extent should the orderly arrangement of the combinations in tables be utilized?
7. Should pupils be required to learn combinations beyond 9×9? If so, how many 12's and how many 15's should be included? Should any others beyond the 10's be required?
8. Which should come first, the short or the long form for division examples requiring carrying and having one-figure divisors? Should both forms be learned by all pupils or by any pupils?
9. What is the best gradation of examples in the division of integers?
10. What is the relative importance and value of the "apparent" rule and the "increase-by one" rule in estimating quotient figures in division.
11. To what extent should practice in the operations with fractions be restricted to those which have been shown to have the greatest social utility?
12. What, if any, rules for divisibility should be taught?
13. What is the best method for teaching pupils to find least common denominators when two or more denominators have one or more factors in common?
14. What is the best way to lead pupils to understand that they can divide by a fraction by multiplying by its reciprocal?
15. Should the important facts of denominate measures be presented in tables or should these facts and relationships be acquired one by one as needed? If the later plan is followed, should the facts of a table eventually be assembled and viewed as an integrated whole by the pupils?
16. What is the relative importance and value of teaching decimal fractions as an extension of our system of decimal notation beyond a decimal point, and as a unique way for writing common fractions whose denominators are integral powers of 10?
17. To what extent should the arithmetic of the elementary school include a treatment of the subject of significant figures?
18. What are the best checks to use for the various fundamental processes? Should pupils' textbooks contain answers?

19. What are the best answers to the many questions currently raised regarding the proper grade placement of arithmetic topics?
20. What is the best use which the elementary school can make of the theories and recommendations of so-called "progressive" education in teaching arithmetic?

On many such questions there has been some research. On all, there have been expressions of opinion. The profession is replete with persons having strong biases on these issues. Some of these persons have written books,—books for teachers and books for pupils.

Five years ago the National Society for the Study of Education, assembled in convention here in Atlantic City, presented the report of the Society's Committee on Arithmetic, the Twenty-Ninth Yearbook. The reception of the Twenty-Ninth Yearbook varied all the way from marked enthusiasm to downright disdain. It is probable that all of us who are interested in arithmetic have read this Yearbook. Many of us have made considerable use of its chapters; some of us contributed to its contents. It contains here and there the fruits of worthwhile pieces of research. In its chapters are found the conclusions of a number of persons who have experimented with and deliberated upon several rather isolated and fragmentary phases of the subject. The speaker shares the opinion of many others that the Yearbook was a valuable contribution.

It will be agreed, however, that there are many important aspects of arithmetic teaching that demand a kind of unified expert attention. A few hints as to what these are have been given in the foregoing list of questions. We need more research—much more research. We need a co-ordinated program of research which shall be directed and participated in by the most capable persons in this country. We need a National Committee on Elementary School Arithmetic. We need a committee of leaders who can forget their prejudices and their commercial ties, if any, long enough to produce an Arithmetic Guide for the teachers, the supervisors, and the textbook writers of this country. This Committee should perform a four-fold function.

First, it should assemble the reports of research projects already completed, evaluate the findings, and make them available to the busy educational practitioners who need them. Frequently, interpretation of results will be necessary and applications to school-

room situations will have to be indicated in a rather specific and concrete manner. In many instances, the Committee will find it necessary to serve as a jury to determine the significance of pieces of research and to point out their values.

Second, it should make a thorough-going survey of research in progress and establish itself as a kind of central clearing house and advisory service that undesired duplication may be avoided, that the most pertinent problems may be attacked, and that the results may be most speedily integrated into the developing program and placed in the hands of the educational public. Here again, an evaluation of findings and judgment as to worth will be required.

Third, it should plan a more extensive research program which shall seek the answers to questions yet unanswered. The Committee would co-ordinate a nation-wide attempt of individuals, groups of individuals, and institutions to attack unsolved problems, the like of which has not yet been known. Many research projects already undertaken have yielded results uncertain and of doubtful value. They need to be repeated on a larger and a better planned scale.

Fourth, to questions which research fails to answer the Committee should be daring enough to propose answers which seem to be in harmony with accepted principles of educational psychology and to be founded upon good judgment. Such questions probably will always exist. They can not be avoided by the classroom teacher and the supervisor. The Committee should assume a position of positive leadership with respect to such questions, admitting in each case that the position is founded upon opinion rather than upon ascertained fact, that it is tentative and therefore subject to later modification, but that it seems to be temporarily, at least, the best way out of the present muddle.

Such a Committee and its program will require money—a considerable sum. I hesitate to suggest the amount. But I recall reports current to the effect that the well known National Committee on Mathematical Requirements received from the General Education Board sixty thousand dollars to finance its work and publish its report.[1] I have in mind an enterprise no less extensive and surely no less worthy than that of the National Committee.

[1] A letter from W. W. Brierley, Secretary of the General Education Board, under date of February 14, 1935, states that the total of these appropriations was approximately $60,800.

What Research Reveals on Proper Drill Content of Elementary Arithmetic[1]

By G. M. WILSON

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DRILL MAY be profitably regarded as a key word in the topic assigned me for discussion. Drill implies material useful enough to justify mastery. The following drill *criteria* are proposed:

First: If the material is not *useful* enough to be needed often in daily life, it is not drill material.

Second: If it is drill material, only one response is correct. Therefore *exact reproduction* is required. Therefore only 100% results are acceptable.

Third: For drill results, the load must be *small enough* to permit success. Success is essential. It provides motivation. It prevents twists of personality. If the load is too large, drill purposes are defeated.

Thus a set of reasonable drill criteria forces conclusions in favor of a small amount of thoroughly useful material perfectly mastered, as the correct objectives of drill in Arithmetic.

The teaching steps in a drill technique[2] point in the same direction. The following may be noted as the essential steps in drill technique:

1. Interest, motivation,—this is the first step in successful drill. Without interest, failure is assured.
2. Thorough understanding is a second important step. Drill must keep well behind meaning.
3. There must be organization of the drill material, such as, provision for sizable units, etc.
4. The laws of memory should be observed such as, repetition with attention, etc.
5. There must be review, use, application.

Interest is not probable when the load is too heavy. Meaning will not be apparent, if the material has little or no use in life outside of school. And thus the recognition of an adequate drill technique forces us in the direction of a small amount of useful arithmetic perfectly mastered.

[1] Read at the annual meeting of the National Council of Teachers of Mathematics at Atlantic City, N.J. on Saturday morning February 23, 1935.

[2] Third Report on Evaluation of Instruction, Department of Classroom Teachers. National Educational Association. July 1926.

The requirements of mental hygiene[3] would force us in the same direction. Mental hygiene in school work requires among other things, (1) a sizable task, (2) the task self chosen, (3) a plan developed by the pupil or with his cooperation and approval, and (4) success.

What is the sizable load in arithmetic, useful enough to require 100% mastery?

If we are to find an answer to this question, where can we look except to usage,—child usage in the beginning stages, adult usage as the limit? To our amazement the answer is surprisingly obvious and simple.

Children count. They occasionally add or subtract. But research shows that small children learn needed numbers as well or better without drill.[4] It is therefore recommended that all systematic drill be deferred to grade three.

Older children need some multiplication, as well as counting, addition, and subtraction. If left without systematic teaching, through grade five, they will learn counting, addition, subtraction and multiplication almost as well as if taught in the traditional school,[5] which of course is much below a proper drill standard. These children do not learn division, since apparently they have little or no use for it.

Adults do not need much division, but they need some. Long division, therefore, is entered as drill material, but there is no hurry about it, grade 5 or grade 6 at the earliest.[6]

Adult usage is 90% covered by the four fundamental processes, their frequency in order being multiplication, addition, subtraction, division. Most adult usage of the fundamentals is in simple form. Multiplications are largely by one or two place multipliers. Most

[3] Burnham, William H. The Normal Mind. Appleton, 1924.

[4] Myers, Garry C. Errors in Arithmetic. The Plymouth Press, 1924. Wilson, G. M. New Standards in Arithmetic. Journal of Educational Research, 22: 351–360, December 1930. Buckingham, B. R. and MacLatchy, Josephine. Number Abilities of Children Entering Grade One, 29th Yearbook of the National Society for the Study of Education, pp. 473–519. Woody, Clifford. Arithmetical Background of Children. Journal of Educational Research, 24: 188–201, October 1931.

[5] Berman, Etta. Master's thesis, Boston University, School of Education, Aug. 1935.

[6] Washburn, Carlton. Arithmetic Grade-Placement. Investigation of the Committee of Seven. Educational Research Bulletin, November 23, 1932, Ohio State University. (See also Journal of Educational Research, March 1931).

additions are two and three place, with few addends. Subtractions are largely making change. Divisions are few and simple, the divisor 12 looming large.

Beyond the fundamentals there are the processes of percentage and interest. These are not child processes. They are not much used by adults. But they are simple and convenient. They can be included in reasonable form in a drill program, but should be defereed for mastery to grade 7 or grade 8.

Fractions if developed to the extent that is common in current texts, would be better entirely omitted from the drill program. Most useful fractions will be learned better through well-planned informational units.[7] This procedure will avoid the fearful confusion resulting from the usual teaching of fractions. One current author carries unit analysis of fraction[8] to such limits as to find 55 specific skills in division of fractions alone. Another author discovers 58 specific types in subtraction of fractions.[9] Neither author is concerned over social values of such work. These are illustrations, pure and simple, of academic insanity. Useful fractions involve no such complications. The fraction one-half alone makes up 60% of adult fractional usage. Halves, with fourths and thirds make up 90%. Few other fractions are needed, occasionally eights or twelfths in special usage. Beyond this (except in specialized vocational usage), reading knowledge only is needed. And, in life, different fractions are seldom used in the same example.[10] Common denominators, if needed, are apparent by inspection.

Decimals, aside from United States money call for reading knowledge only. Beyond this a small amount of informational attention should meet all requirements. All process drill on decimals should be omitted. Decimals are not replacing common fractions in daily use. When decimals do come into daily usage with new machinery or new recording devices, reading knowledge only is required. Attempts to force decimals beyond this end in failure. Attempts through a century to force decimalization in the use of

[7] Harap, Henry. Sixth Grade Units for Applying the Arithmetic of Decimals. No. 23, Curriculum Laboratory, Western Reserve University, 1934.

[8] Knight, F. B. Third Yearbook, Department of Superintendence 1925 pp. 66–68.

[9] Brueckner, Leo. J. A Technique for Analyzing the Distribution of Drill in Fractions. Journal of Educational Methods. 7: 352–358, May 1928.

[10] Dalrymple, Charles O. The Fractions of Business and Life. Doctors Dissertation, Boston University, 1933.

the metric system have ended in failure. In common usage, there is no more decimalization under the metric scheme of weights and measures, than under the customary scheme. The uses of decimals on a drill basis regardless of scheme or system are confined to highly trained specialists, and the training of these specialists is no part of the drill program of the elementary schools.

Child usage finds nothing of drill value in denominate numbers as developed in current texts in arithmetic, nor does adult usage. Some of it finds a proper place in the cyclopaedia. Much of it is purely academic, developed at the desk of a college professor, carried forward traditionally, and having no counter-part in life outside school.

Let us be specific. Does one buy any commodity by a table? He does not, he buys in one unit, the customary trade unit. One buys gasoline, e.g., by the gallon. Yet a recent text would have a pupil of the sixth grade figure the cost of gas in terms of gals., qts., pints, and gills, after performing a compound subtraction. We would question the sanity of anyone who went about it that way in real life. We buy butter by the pound, not in tons, cwt., lbs., and ozs. We sell wheat by the bushel not in bus., pks., gals., and qts. But in all cases, we need the unit only after we know the commodity, not before. Drill is not the profitable method of approach to compound numbers, or weights and measures. In general, only such knowledge is retained in this field as comes through experience.[11]

No one, not academically unbalanced would recommend drill in the elementary school in the mere manipulation of figures, such as factoring, least common multiple, greatest common denominator, ratio and proportion, and square root.

The profitable approach to business is not through drill.

It would appear, therefore, that drill on the elementary level should aim at 100% mastery of the four fundamentals, very simple fractions, percentage and interest. I've seen no better plan than that suggested in the Fourth Yearbook of the Department of Superintendents:

> Mastery of addition and subtraction in grade 3; mastery of multiplication and short division in grade 4, review addition and subtraction; mastery of long division

[11] Wilson, Dorothy W. What Weights and Measures Facts are Retained in the Memory and Why. Master's Thesis, Boston University, School of Education, in preparation. Also unpublished researches of the writer.

in grade 5, review addition, subtraction, and multiplication; mastery of percentage and interest in grade 6 or 7; fractions on a mastery basis limited to halves, fourths, thirds, eighths, and twelfths, with crossing denominators limited to halves and fourths.

This program of Drill mastery represents a sizable load of arithmetic materials useful enough to justify 100% mastery, i.e. drill. It is supported by research.[12]

May I add that I find myself in agreement with others who see in elementary mathematics other values and other opportunities than those realized through drill mastery of the more useful processes. This paper makes no attempt to point out these values and opportunities nor to indicate the procedures by which they may be best realized. It is proper, however, to indicate that, in large measure, these values and opportunities have, for most pupils, been defeated by carrying the drill procedure beyond its proper limits. Reduction of time on drill does not necessarily mean reduction in total time for mathematics; it does call for a better direction of much of the school time devoted to mathematics.

[12] See summary of earlier research in What Arithmetic Shall We Teach, by Wilson. Houghton, Mifflin Co. Boston, 1926.

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A Psychological Analysis of Students' Reasons for Specific Errors on Drill Materials in Plane Geometry*

By LYLE K. HENRY
University of Iowa

I. PROBLEM: The aim of this investigation was to determine why students made certain specific errors on drill exercises in plane geometry. The fact that the pupil does not know the right answer is no reason at all, psychologically. When an incorrect answer is given, the response is due to certain factors just as surely as when the right answer is given. The aim of the study was to discover these factors, in so far as possible, by individual conferences with the pupils on specific errors made. It should be emphasized that the *cause* of specific errors was the point of investigation rather than the type of material in which errors occurred.[1]

II. *Procedure:* The data for the study were collected from an Iowa high-school class of twenty-seven tenth-grade students of average intelligence and geometrical ability.[2]

The drill exercises, prepared by Welte and Knight,[3] were used as part of the course materials. These materials were valid for the course in that they contained the minimum essential theorems specified by the National Committee on Mathematics Requirements and six prominent texts, including the one used by the class. One drill was given each Friday for twenty weeks of the school year. Twenty minutes were allowed to complete each drill

* The writer expresses indebtedness to Dr. F. B. Knight who directed this study as a Masters Thesis.

[1] The types of material in which errors most frequently occur as well as the reduction of plane geometry to main ideas have been worked out by H. D. Welte. See Welte, H.D., *A Psychological Analysis of Plane Geometry*, University of Iowa, Monographs in Education, Jan. 1, 1926; Welte, H. D., *A Critical Study of Errors in Plane Geometry*, Ph.D. Thesis, University of Iowa, 1929.

[2] Intelligence Quotient on Otis Group, Form B, range-89–122, average—105; Iowa Academic Test in Geometry, average score of class B schools—11.78, this class —12.65.

[3] Welte and Knight, *Standard Service, Geometry Work Book*, Chicago: Scott, Foresman & Co., 1929.

and the remaining twenty minutes of the period were used to check errors and discuss points of interest. The drill materials were then collected by the instructor and were in the hands of the pupil at no other time.

The errors were tabulated for each drill and the five most frequently missed exercises of each drill (each drill contained 20 exercises) were selected for study. This process netted 945 errors which were significant because of their frequency and upon which the results were based.

Interviews were held immediately (on the afternoon following the interview or early the next week) with all students who made errors on the five most frequently missed exercises of that drill. The investigator was careful to establish and maintain proper rapport with the pupil, explaining that the purpose was to find out how pupils learned geometry and how it might be taught more effectively. Students were asked why they missed the exercise, what part bothered them most, and why they gave their particular answer. The plan was successful in revealing in most cases what seemed to be the pupil's difficulty. These reports were then compiled to obtain types of causes of error.

III. *Results and Discussion:* The causes of error as determined by the procedure described are presented below.

TABLE 1

Type No.	*Cause of Error*	*Frequency*	*Percent of Total Error*
1	Confusing or mixing rules	206	.22
2	Inability to define terms	147	.16
3	Failure to identify data in figure	131	.14
4	Misinterpretation of statement	128	.13
5	Faulty perception of statement	116	.12
6	Forgetting the rule	98	.10
7	Arithmetical error or disability	88	.09
8	Misapplication of idea	24	.03
9	Carelessness or haste	7	.01
		945	100%

These type causes will be discussed in regard to their nature and the typical material in which they occurred.

TYPE NO. 1—CONFUSING OR MIXING RULES

The term "rules" refers in most cases to theorems, but it applies to axioms and corollaries as well. By confusing, or mixing, the writer means that the students had in mind a theorem or rule that

dealt with a rather general or similar situation, but was not specifically adapted to the problem at hand. The situations most susceptible to this error were congruency of right triangles, definitions of terms and characteristics of polygons. In the case of congruency of triangles most students knew that there were several similar theorems and proceeded to make a superficial choice, without regard for the data in the figure. In making the case study of congruency theorems it became apparent that the students had made their decisions by looking at the figure as a whole.

Type No. 2—Inability to Define Terms

This type of cause, which is self explanatory, was responsible for a large number of errors in commonplace and seemingly simple situations. For example, 33 errors were due to the inability of students to tell the difference between two kinds of polygons, which were usually quadrilaterals. Other situations prompting several errors were the converse of theorems—25 errors; proportion—17 errors, types of triangles—16 errors, types of angles—15 errors, and analyzation of statements—14 errors.

Type No. 3—Failure to Identify Data in Figure

This type of cause was most frequent where interpretation of constructions or relative values was called for. Students failed to recognize a right triangle if the figure rested on a side other than the side on which they were accustomed to seeing it rest. When students were asked to interpret or use data as obtained from a figure, their error was usually due to inability to recognize the particular part in question.

Type No. 4—Misinterpretation of Statement

This point covers general types of reading deficiency and disability and, as one would suppose, most errors of this kind occur where the situation requires an analysis of the statement. In this analyzation the student was asked to tell what was given in a statement, or what was to be proved. Failure in this analysis resulted when the comprehension of the reader was poor.

Type No. 5—Faulty Perception of Statement

Where this type of cause occurred there was no figure accompanying the statement and the student was unable to formulate a

mental picture, or produced the wrong picture. Other things being equal an average amount of visual and mental imagery would have resolved the difficulty. This type of error occurred most frequently in dealing with types of angles and analysis of statements.

Type No. 6—Forgetting the Rule

This type of cause usually appeared in an answer in the form of a "hunch," a guess or a complete omission. Let us not forget here, that guessing is not the cause of error, but is the result of incomplete or completely forgotten material. This cause showed up most frequently in regard to finding the sum of the interior angles of a polygon. Students usually forgot the "x 2 rt. angles" part of the formula and, as a result, omitted the 2 or substituted another number, which resulted in a wide range of answers. Case studies show that the errors attributed to this cause were not prompted by the inability to define the type polygon, as one might suppose on first thought. Too, this error appeared when the figure of the polygon appeared with the statement.

Type No. 7—Arithmetical Error or Inability

The inability to handle the radical, when found in one of the dimensions of a right triangle, was the chief difficulty. Fifty-two of the 88 errors in arithmetic were due to difficulty in working with the radical sign. Twelve errors were due to inability to work proportion, while the other errors were due to miscalculation in mental arithmetic.

Type No. 8—Misapplication of the Idea

Although it may seem to the reader that the 24 errors making up this type might have easily been classified under Type No. 4 or No. 5, the evidence shows that there was a different factor operating. Half of the 24 errors were due to the misapplication of the term "converse." The students knew the meaning of the term but made errors in the application. The other 12 errors were made on a True-False statement that "an arc of a circle is any curved line." Here, again, the students understood the concept but failed to use it correctly. This type of cause might be termed as faulty reasoning.

Type No. 9—Carelessness or Haste

In only one exercise was the error assigned to carelessness or haste. There were, no doubt, other cases but those instances re-

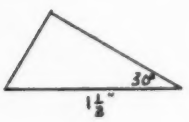
solved themselves into a more logical form of explanation. At any rate, the writer feels that the assigning of numerous errors to haste or carelessness would be a sort of regression in constructive explanation.

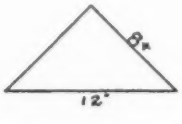
The exercise in question dealt with the parts of a figure and, at the outset, one might think that the cause of the error was Type No. 3—(Failure to identify data in the Figure). However, case reports, individually given, show that cause No. 3 has in turn an underlying cause in isolated instances.

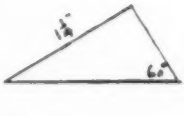
IV. *Recommendations:* On the basis of the case studies of error made in this investigation it seems likely that proper instructional attention to the following points, along with adequate instruction on other aspects, would eliminate a large percentage of the types of errors encountered in the study.

1. Teach congruency of right triangles as a unit rather than as three or four separate consecutive theorems. Use a comparative drill exercise such as the following:

According to the direct application of the data given, have the student tell why the pairs of right triangles are congruent. Pairs should then be presented in which the triangles appear in different positions as well as in groups of figures containing non-congruent triangles.







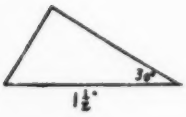


Fig. 1

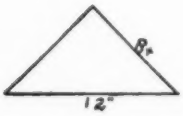


Fig. 2

Fig. 3

2. Provide practice in reading data from figures and interpreting figures. Give students a rich variety of experiences in perception of figures and relationships in all positions and sizes and with various notations. Do not continually set figures on one side and no other. Students fail to transpose principles and proofs that are learned in one setting only.

3. Provide training and practice in the critical reading and analysis of problems and exercises. Our data show that many students simply cannot tell what is given or what is to be proved, aside from that which we consider the principal task, namely, providing the proof after the data and problem are known.

4. Make clear the meaning and restrictive characteristics of such terms as corollary, proposition and axiom, by strict definition such as the following: An axiom is a basic, established arithmetic fact; a postulate is just as basic but is a geometric fact. A proposition, theorem or problem is a formal statement of a truth to be demonstrated, and the corollary is a deduction, consequence, or additional reference from the proved proposition. A corollary should, therefore, always be associated with a definite theorem.

5. Use a drill exercise in teaching the meaning of the term "converse" to avoid misconceptions of it. Two general types of misconceptions appeared: (1) That it means the opposite. For example, the converse of a statement including alternate-exterior was written—alternate-interior. (2) That it means rearranging the sentence, or stating it "backwards." A drill exercise similar to the following would eliminate a number of these errors:

Statement	*Converse*
1. *If it rains, it will turn cooler.*	1. *If it turns cooler it will rain.*[4]
2. *If a man is rich he is happy.*	2. *If a man is happy he is rich.*[4]
3. *If my eyesight is good, I see clearly.*	3. *If I see clearly my eyesight is good.*
4. *If a straight line figure has three sides it is a triangle.*	4. *If a figure is a triangle it has three straight line sides.*

6. Take proper instructional care of such details as the following:

a. In referring to supplementary angles, point out the fact that angles do not have to be adjacent in order to be supplementary.

b. Designate and refer to angles by number, as $\angle 1$ angle 1, instead of $\angle AOB$ angle AOB.

c. In the study of the characteristics of quadrilaterals, correct or avoid the misconception that a rectangle must be longer than it is wide.

[4] Point out that the converse of a statement is not always true. See Hawkes, Luby and Touton's New Plane Geometry. Ginn, 1929, p. 75.

7. Before starting the application of the Pythagorean theorem, review the arithmetical processes involving the use of the radical sign.

8. Use a "negative program" for all basic theorms, that is, point out "exceptions," differences between similar theorems and usual pitfalls of the situations.

9. Remember, above all, that the insight of the ordinary student is much below that of the instructor. Therefore, steps and procedures which are logical to the instructor may not be logical to the student. This means that more attention must be given to the *so-called* simple concepts, the misunderstanding and ignorance of which have been responsible for a majority of the errors found in this study.

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The Prediction of Pupil Success in High School Mathematics

By HARL R. DOUGLASS

Professor of Secondary Education, University of Minnesota

FAR MORE frequent than any other type of investigation relating to secondary school mathematics has been that which concerned itself with the relationship between scholastic success and other factors—intelligence test scores and their derivatives, I.Q. and M.A., and previous school marks being employed most frequently.

Information concerning the degree of correlation between various factors of this sort is valuable not only for the light it may throw upon the problem of the community of function between the study of mathematics and general mental ability, various special subjects, general school ability and the like, but it is particularly valuable in evaluating the variables studied as possible prognostic bases in connection with homogeneous grouping, educational guidance and similar applications of individual psychology.

Space would not permit here a review of each of these studies. It must suffice to present here in tabular form the coefficients of correlation between scholastic success in high school mathematics and a number of various predictive bases.

Space is not available here for the discussion of all the inferences which may be drawn from the foregoing data. It should be remembered that in addition to the essential relationship between two or more variables, certain other factors affect the magnitude of a coefficient of correlation and in fact any mathematical means of measuring co-relationship. Most prominent of those which must not be lost sight of are three. First, the size of the coefficient is dependent upon the homogeneity of the group upon which it is based. The more heterogeneous the group, the greater the coefficient obtained, other things remaining equal. Second, obtained coefficients of correlation are smaller than those which would be found, if perfectly reliable measures of the two variables are employed. Test scores and teachers' marks are never completely reliable. Third, the criteria of achievement are not likely to be fully valid—marks often being measures of docility, promptness, personality, attend-

ance, diplomacy, etc., and test scores as well as marks measuring not all of the desirable objectives of instruction and of course, not in proportion to their relative importance, which indeed is not known with any notable degree of accuracy. Because of these and other reasons it scarcely need be pointed out that comparison of coefficients based upon the same pupils may be made with much more safety than may be done with coefficients based upon different groups.

While space was not available for presenting for the purpose of comparison with coefficients given here the many coefficients of correlation between the various predictive factors listed in the foregoing table and achievement in the subjects other than mathematics, many of the investigators quoted from have supplied them. While there is not agreement among the authors of reports, it is fair to say that achievement in mathematics can be predicted with almost the same degree of accuracy as the average of other "academic" subjects, and definitely with more accuracy than achievement in art, science, physical education, household or industrial arts. The various factors correlate less definitely and less closely with mathematics than total high school achievement, and tend to also correlate slightly less closely with achievement in mathematics than with achievement in English, Latin, and history. In mathematics as in other subjects, correlations tend to become smaller with advance in grade and in size of pupil.

Studies of relationship between various factors and achievement in seventh or eighth grade arithmetic are not quoted here. Useful studies in this field are reported by Kefauver (1929) and Brooks (1925) (See bibliography at end of this article.) Obviously space is not available even to mention the numerous studies, probably over a hundred of them, of the relationship between general or average achievement in the high school, although the measures of achievement employed include marks or test scores in mathematics. Summaries of and bibliographical references to many of these studies are given in Brooks (1929), Symonds (1927), Ross and Hooks (1930), and in most of the general treatises on guidance and on intelligence testing.

In addition to the coefficients presented there are certain other findings which may be briefly presented here. In algebra, Short (1927) reports that there are slight chances of success for pupils with Terman I.Q.'s less than 85. McCuen (1930) found that the

arithmetic and number series sections of the Terman Group Test of Mental Ability each correlated about as highly with Douglass Algebra Test scores as did the scores on the entire test. Haddock (1927) reported that pupils receiving higher marks in algebra had been previously found to have a higher E.Q. in the Stanford Arithmetic test and that each successive lower category of marks included pupils of progressively lower E.Q.'s in arithmetic. Orleans (1934), who, obtained a coefficient of .54 between I.Q.'s (Otis S.A.) and teachers' marks in algebra, computed bi-serial r's between failing-passing and his prognostic algebra test and found them to range from .66 to .81. French (1927) found it was more difficult to predict achievement in algebra than in English and Latin. Madsen (1922) also found a correlation between I.Q. (Army Alpha) and achievement in algebra in two large Omaha high schools as evidenced by the fact that the average I.Q. of those failing was 111 for boys and 99 for girls. Cooke and Field (1932) found that achievement in geometry could be predicted no better by the Stanford Achievement Arithmetic Test and Detroit Advanced Intelligence Examination than by the latter alone. Mitchell (1935) reported that of pupils in West Newton, Pa. less than 1 in 10 of those with I.Q.'s (Terman Group Test of Mental Ability) greater than 100 failed in Algebra, while of those with I.Q.'s of 92–99, approximately one-fourth failed; of those of I.Q.'s of 78–90, one-half failed, while practically all of I.Q.'s less than 78 failed.

In geometry, Bi-serial r's between passing-failing and scores on the Orleans Prognostic Geometry Test were reported by Orleans (1934), ranging from .51 to .84. Crafts (1923) studied two groups of failures in plane geometry of 62 and 45 pupils respectively, in East High School at Rochester, N.Y., and reported that these failures made scores on the Terman Group Test of Mental Ability corresponding to a mental age two years less than that for those who passed.

Evidence that having foreign born parents is associated to a slight degree with poor achievement in arithmetic and algebra is presented by Koenig (1928) based on the records of junior high school pupils at Masontown, Pennsylvania. Flemming (1925) was not able to demonstrate any relationship between achievement of girls in mathematics and temperament as measured by an adaptation of the Downey Will Temperament Scale, either at the junior high school or at the senior high school level. Rogers (1918) re-

ported a correlation of .52, .38, and .47 between algebra ability and geometry ability as measured by her prognostic tests. This is in close correspondence with the conclusions of Lee and Lee (1931) who report coefficients of .55 and .57 between the Lee Test of Algebra Ability and the Lee Test of Geometry Ability.

General Conclusions

1. Achievement in algebra and geometry may be predicted with a fair degree of accuracy only.
2. Achievement cannot be predicted satisfactorily from any one variable for the purposes of homogeneous or ability grouping or definite advice relative to taking or not taking algebra or geometry.
3. Achievement is best predicted by a combination of the following variables—a good prognostic test, I.Q. and average mark in previous year or two years of school work.
4. While the relative order of merit is not clearly established it would seem that the better variables uses for prediction of success have something the following rank in order of validity though there is little choice between the variables a and b; variables c and d; and variables e, f and g.

   (a) Good prognosis test
   (b) Average mark in previous year
   (c) I.Q.
   (d) Previous teacher's estimate of mathematics ability
   (e) M.A.
   (f) Achievement test or mark in previous year's work in math.
   (g) Chronological age
   (h) character trait rating

Probably almost as good a basis for prediction as variables e, f, g and h is socio-economic status as determined by the Barr or Sims scales.

BIBLIOGRAPHY

Anderson, Harrison H. *Ability to Solve Written Problems in Algebra as a Prognosis of Success in First Semester Plane Geometry.* Master's thesis, Indiana University, 1929. Abstract in Foster (1933 this list).

Bright, Ira J. "The Intelligence Examination for High School Freshmen." *Journal of Educational Research,* 4: 44–55, June, 1921.

Brooks, Fowler D. "Sectioning Junior High School Pupils" *Journal of Educational Research,* 12: 359–369, June, 1925.

Brooks, Fowler D. *Psychology of Adolescence.* Houghton Mifflin Co., 1929, pp. 553–590.

Brown W. "An Objective Study of Mathematical Intelligence" *Biometrika* 7: 352–367; 1909.

Burris, W. P. *Correlations of the Abilities Involved in Secondary School Work Edited by E. L. Thorndike.* Columbia University, Contributions to Education, Vol. XI, No. 2, 1903.

Cooke, D. H. and Pearson, J. M. "Predicting Achievement in Plane Geometry." *School and Science and Mathematics* 38: 872–878; 1933.

Cooke, D. H. and Fields, C. L. "Relation of Arithmetical Ability to Achievement in Algebra and Geometry." Bibliography. *Peabody Journal of Education* 9: 355–61; May, 1932.

Crafts, Lillian. "Causes of Failure in Plane Geometry as Related to Mental Ability" *The Mathematics Teacher*, 16: 481–92, December, 1923.

Crathorne, A. R. *The Theory of Correlation Applied to School Grades in the Reorganization of Mathematics in Secondary Education.* Ch. X. The Mathematics Association of America, 1923.

Dickinson, C. E. "Reading Ability and Scholastic Achievement." *School Review* 33: 616–26; October, 1925.

Dickter, M. R. "Predicting Algebraic Ability" *School Review* 41: 604–6; October, 1933.

Elder, Harry E. "Percentile Rank in Intelligence as a Prognosis of Success in Algebra." *School Review* 34: 543–46; September, 1926.

Flack, W. S. *Investigation of Mathematical Ability in the Classroom.* Forum of Education, 4: 44–6. February, 1926.

Flemming, Cecile W. "A Detailed Analysis of Achievement in High School" Teachers College, Columbia University, Contributions to Education, No. 196, 1925.

Foster, I. O. "Implications of Some Unpublished Research Studies in High School Mathematics" *Bulletin of the Department of Secondary Education of the N.E.A.* 2: 29–31; February, 1933.

French, John M. *The Value of Tests and Teachers' Marks as a Means of Predicting the Success of Pupils in High School.* Master's thesis, University of Chicago, 1927. Abstract in Bulletin of Department of Secondary School Principals of the N.E.A., No. 24, January, 1929, pp. 61–65.

Grover, G. C. "Results of an Experiment in Predicting Success in First Year Algebra in Two Oakland Junior High Schools." *Journal of Educational Psychology* 23: 309–14; April, 1932.

Haddock, Nellie P. Use of Stanford Achievement Test to Indicate High School Success. School and Society, 25: 114–16, January 22, 1927.

Jordan, R. M. "Correlations of Four Intelligence with Grades" Journal of Educational Psychology 13: 419–29; September, 1922.

Kefauver, G. N. "The Validity of Bases for Forming Ability Groups" Teachers College Record 31: 99–114, November, 1929.

Kelley, Truman L. "Educational Guidance" Teachers College, Columbia University, Contributions to Education, No. 71, 1914.

Kertes, F. "Ability Grouping in High School." *Mathematics Teacher* 25: 5–16; January, 1932.

Koenig, Wm. H. *Partial Correlations Showing the Relationship Between Success in School and Certain Factors Outside of School.* Master's thesis, Pennsylvania State College, 1928. Abstract in Penn State Studies in Education, No. 2, pp. 73–4.

Lee, J. M. & Hughes, W. H. "Predicting Success in Algebra and Geometry." *The School Review* XLII: 188–97; March, 1934.

Lee, D. M. & Lee, J. M. "Some Relationships Between Algebra and Geometry" *Journal of Educational Psychology* 22: 551–60; October, 1931.

Lee, J. M. & Lee, D. M. "The Construction and Validation of a Test of Geometric Aptitude," *Mathematics Teacher* XXV: 193–203; April, 1932.

Madsen, I. N. "The Contribution of 9 Intelligence Tests to Educational Guidance in High School." *School Review* 30: 692–701; November, 1922.

McCuen, Theron L. "Predicting Success in Algebra." *Journal of Educational Research* 21: 72–4; January, 1930.

Mitchell, Claude, "Prognostic Value of Intelligence Tests." *Journal of Educational Research* 28: 577–81, April, 1935.

Ohlson, David, "School Marks vs. Intelligence" *Educational Administration and Supervision* 13: 90–102, February, 1927.

Orleans, Joseph B. "A Study of Prognosis of Probable Success in Algebra and in Geometry." *Mathematics Teacher* 27: 165–180 and 225–246; April and May, 1934.

Rogers, A. L. *Tests of Mathematical Ability and Their Prognostic Value.* Teachers College, Columbia University, Contributions to Education, No. 89, 1918.

Ross, C. C. *The Relation between Grade School Record and High School Achievement.* Teachers College, Columbia University, Contributions to Education, No. 166, 1925.

Ross, C. C. & Hooks, N. T. "How Shall We Predict High School Achievement" Journal of Educational Research, 22: 184–187; October, 1930.

Seltzer, Ray L. *Correlations of Certain Abilities of Pupils with Success in Algebra.* Master's thesis, Pennsylvania State College, 1932. Abstract in Penn State Studies in Education, No. 8. pp. 65–6.

Shewman, W. D. "Study of Intelligence and Achievement of June, 1925 Graduating Class of Grover Cleveland High School of St. Louis." *School Review* 34: 137–45; 219–227; February and March, 1926.

Short, Vivian. *A Study of the Relation of Intelligence and Achievement in the Specific Skills of Algebra.* Master's thesis, University of Minnesota, 1927. Abstract in *Bulletin of the Department of Secondary School Principals of the N.E.A.*, No. 47. May, 1933, pp. 47–8.

Symonds, Percival M. *Measurement in Secondary Education.* Macmillan, 1927, Ch. XIX and XX.

Unzicker, Samuel P. "Teachers Marks and Intelligence," *Journal of Educational Research* 11, 123–37, February, 1925.

Weglein, D. E. *The Correlation of Abilities of High School Pupils.* Johns Hopkins University Studies in Education, No. 1, 1917.

Winegarden, J. H. *The Relation of Science in Mathematics to Success in Physics and Chemistry in High School.* Master's thesis, Stanford University, 1929.

TABLE I

Coefficients of Correlation between Success in Ninth Grade Algebra or General Mathematics and Intelligence Aptitude Test Scores, Previous Scholastic Achievement, and Combinations

Author	*Date*	*Criterion*	*Variable*	*N*	*Place*	*Correlation*
			INTELLIGENCE			
Bright	1921	Final Exam. Scores	I.Q. (Terman)	142	Leavenworth, Kan., H.S.	.44
Bright	1921	Teachers' Marks	I.Q. (Terman)	142	Leavenworth, Kan., H.S.	.50
Dickter	1933	Breslich Algebra Test	I.Q. (Otis Group)	83	Jr. H. S., Upper Darby, Pa.	.54
Grover	1932	Tests	I.Q. (Terman)	71	Oakland Jr. H. S.	.48
Kertes	1932	1st Semester Teachers' Marks	I.Q. (Terman)	114	Perth Amboy, N.J.	.56
Lee & Hughes	1934	Pol. Res. Bureau Alg. Test	I.Q. (Kuhlman Anderson)	197	Three Calif. Jr. High Schools	.56
Lee & Hughes	1934	Pol. Res. Bureau Alg. Test	I.Q. (Terman)	197	Three Calif. Jr. High Schools	.47
Lee & Hughes	1934	Teachers' Marks	I.Q. (Kuhlamn Anderson)	197	Three Calif. Jr. High Schools	.48
Lee & Hughes	1934	Teachers' Marks	I.Q. (Terman)	197	Three Calif. Jr. High Schools	.44
McCuen	1930	Douglass 1st Sem. Alg. Tests	I.Q. (Terman)	24	Palo Alto, Calif. H.S.	.47
McCuen	1930	Douglass 1st Sem. Alg. Tests	I.Q. (Terman)	28	Plao Alto, Calif. H.S.	.34
McCuen	1930	Douglass 1st Sem. Alg. Tests	I.Q. (Terman)	32	Palo Alto, Calif. H.S.	.54
McCuen	1930	Douglass 1st Sem. Alg. Tests	I.Q. (Terman)	32	Palo Alto, Calif. H.S.	.42
McCuen	1930	Douglass 1st Sem. Alg. Tests	I.Q. (Terman)	48	Palo Alto, Calif. H.S.	.57
McCuen	1930	Douglass 1st Sem. Alg. Tests	I.Q. (Terman)	116	Palo Alto, Calif. H.S.	.42
Proctor	1921	Marks	Stanford I.Q. (Stanford Binet)		Palo Alto, Calif. H.S.	.46
Seltzer	1932	Semester Teachers' Marks	I.Q. (Otis S.A.)	100	St. Clair, Pa. H. S.	.57
Short	1927	Douglass Diag. Alg. Tests	I.Q. (Terman)	119	Minneapolis High Schools	.38
Short	1927	Douglass Diag. Alg. Tests	I.Q. (Terman)	86	Minneapolis High Schools	.45
Short	1927	Douglass Diag. Alg. Tests	I.Q. (Terman)	45	Minneapolis High Schools	.39
Short	1927	Douglass Diag. Alg. Tests	I.Q. (Terman)	65	Minneapolis High Schools	.67
Short	1927	Douglass Diag. Alg. Tests	I.Q. (Terman)	315	Minneapolis High Schools	.50
Brooks	1929	Marks	Mental Age (Illinois)	73	Baltimore Jr. H.S.	.42
Brooks	1929	Marks	Mental Age (National)	73	Baltimore Jr. H.S.	.30
Brooks	1929	Marks	Teachers' Estimate of Intelligence	73	Baltimore Jr. H.S.	.23
Buckingham	1921	Marks	Army Alpha		Urbana Ill., H.S.	.38
Buckingham	1921	Marks	Terman Group		Urbana Ill., H.S.	.25
Crathorne	1922	Marks	Average of Two Mental Tests			.50

TABLE I—Continued

Author	*Date*	*Criterion*	*Variable*	*N*	*Place*	*Correlation*
Dickenson	1925	Marks	Pressey Mental Survey Scale	149	Logansport, Ind. H.S.	.23
Elder	1926	Teachers' Marks	P.R. (Otis S.A.) plus (Indiana Mental Scale, No. 1)	50	Monticello, Ind. H.S.	.60
Hooks	1928	Marks	P.R. Otis Group	82		.31
Jordan	1922	Marks	Army Alpha Test Score	47	Univ. of Ark. H.S.	.51
Jordan	1922	Marks	Miller Test Score	47	Univ. of Ark. H.S.	.46
Jordan	1922	Marks	Otis Group Test Score	47	Univ. of Ark. H.S.	.43
Jordan	1922	Marks	Terman Test Score	47	Univ. of Ark. H.S.	.44
McCuen	1930	Douglass 1st Sem. Alg. Tests	Terman Intell. Score	32	Palo Alto, Calif. H.S.	.27
McCuen	1930	Douglass 1st Sem. Alg. Tests	Terman Intell. Score	24	Palo Alto, Calif. H.S.	.32
McCuen	1930	Douglass 1st Sem. Alg. Tests	Terman Intell. Score	28	Palo Alto, Calif H.S.	.27
McCuen	1930	Douglass 1st Sem. Alg. Tests	Terman Intell. Score	32	Palo Alto, Calif H.S.	.31
McCuen	1930	Douglass 1st Sem. Alg. Tests	Terman Intell. Score	116	Palo Alto, Calif. H.S.	.31
McCuen	1930	Douglass 1st Sem. Alg. Tests	Terman Intell. Score	48	Palo Alto, Calif. H.S.	.43
Ohlson	1927	Marks	Terman Group Score	476	Everett, Wash. H.S.	.35
Ross	1925	Marks	Terman Intelligence Test	79	Des Moines, Ia. H.S.	.42
Dickinson	1925	Marks	Thorndike Visual Vocabulary Test	149	Logansport, Ind. H.S.	.45
Dickinson	1925	Marks	Thorndike McCall Silent Reading Test	149	Logansport, Ind. H.S.	.43
			PROGNOSTIC AND ACHIEVEMENT TESTS			
Dickter	1933	Breslich Algebra Test	Roger's Test of Math. Abil.	83	Jr. H.S., Upper Darby, Pa.	.64
Grover	1932	Columbia Research Tests	Orleans Prognostic Test	71	Oakland Jr. H.S.	.61
Jordan	1922	Marks	"Abil. Problems" part of Otis Test	47	Univ. of Ark. H.S.	.68
Jordan	1922	Marks	"Hard Oral Directions," part of Army Alpha	47	Univ. of Ark. H.S.	.61
Kelley	1914	Marks	Algebra Aptitude Test	123	Two N.Y. City High Schools	.47
Kelley	1914	Marks	Math. Interest Test	123	Two N.Y. City High Schools	.30
Kertes	1932	1st Sem. Teachers' Marks	Orleans Prognostic Test	114	Perth Amboy, New Jersey	.61
Lee & Hughes	1934	Pol. Res. Bur. Alg. Test	Lee Aptitude Test of Algebra Abil.	197	Three Calif. Jr. High Schools	.62
Lee & Hughes	1934	Pol. Res. Bur. Alg. Test	Teachers Estimate, Math. Abil.	197	Three Calif. Jr. High Schools	.53

Lee & Hughes	1934	Teachers' Marks	Lee Aptitude Test for Alg.	197	Three Calif. Jr. High Schools	.46
Lee & Hughes	1934	Teachers' Marks	Teachers' Est. of Alg. Abil.	197	Three Calif. Jr. High Schools	.59
Orleans	1928–30	Columbia Research Alg. Tests	Orleans Prognostic Test	29–280	Large N.Y. City High Schools	17 r's from 51 to 80 ave. = 63
Rogers	1916	Teachers' Marks	Roger's Math. Abil. Tests	53 girls,	Wadlwigh H.S. (N.Y. City)	.70
Rogers	1916	Teachers' Marks	Roger's Math. Abil. Tests	61 girls,	Horace Mann, H.S., N.Y. City	.88
Rogers	1923	Teachers' Marks	Roger's Math. Abil. Tests	26–115	Ten High Schools	.34 to .76
Ross	1925	Marks	Thorndike McCall Reading Test	79	Des Moines, Ia. H.S.	.31
Ross	1925	Marks	Woods-McCall Arithmetic Test	79	Des Moines, Ia. H.S.	.46
McCuen	1930	Douglass 1st Sem. Alg. Tests	Stanford Arith. Achievement Test	48	Palo Alto, Cal. H.S.	.46
McCuen	1930	Douglass 1st Sem. Alg. Tests	Stanford Arith. Computation Test	48	Palo Alto, Cal. H.S.	.36
McCuen	1930	Douglass 1st Sem. Alg. Tests	Stanford Arith. Reasoning Test	48	Palo Alto, Cal. H.S.	.39
Seltzer	1932	Semester Teachers' Marks	Courtis Arith. Addition	100	St. Clair, Pa. H.S.	.44
Seltzer	1932	Semester Teachers' Marks	Courtis Arith. Multiplication	100	St. Clair, Pa. H.S.	.49
			TEACHERS' MARKS AND OTHERS			
Brooks	1929	Marks	Av. Marks, Grade 6	73	Baltimore Jr. H.S.	.25
Brooks	1929	Marks	Av. Marks, Grade 7	73	Baltimore Jr. H.S.	.45
Brooks	1929	Marks	Av. Marks, Grade 8	73	Baltimore Jr. H.S.	.61
Dickter	1933	Breslich Algebra Tests	Marks in 8th grade arith.	83	Jr. H.S., Upper Darby, Pa.	.61
French	1927	Marks	8th Grade Arithmetic Marks		Martinsville, Ind. H.S.	.14
French	1927	Marks	Stanford Ach. Test in Arithmetic		Martinsville, Ind. H.S.	.32
Kertes	1932	1st Semester Teachers' Marks	8th Grade Arith. Marks	114	Perth Amboy, New Jersey	.63
Ross	1925	Marks	Marks in Each of Various Studies in all Grades	134	New Rochelle, N.Y. H.S.	.08 to .50
Ross	1925	Marks	Average 8th Grade Mark	134	New Rochelle, N.Y. H.S.	.30
Ross	1925	Marks	Arithmetic Mark, Grade 7	134	New Rochelle, N.Y. H.S.	.32
Ross	1925	Marks	Arithmetic Mark, Grade 8	134	New Rochelle, N.Y. H.S.	.39
Ross	1925	Marks	Arithmetic Mark, Grades 7 and 8	134	New Rochelle, N.Y. H.S.	.42
Ross	1925	Marks	Arithmetic Mark, Grades 2–8	134	New Rochelle, N.Y. H.S.	.38
Ross	1925	Marks	English Mark, Grade 7	134	New Rochelle, N.Y. H.S.	.26
Ross	1925	Marks	English Mark, Grade 8	134	New Rochelle, N.Y. H.S.	.34
Ross	1925	Marks	English Mark, Grades 8 and 9	134	New Rochelle, N.Y. H.S.	.35
Ross	1925	Marks	English Mark, Grades 2–8	134	New Rochelle, N.Y. H.S.	.34

TABLE I—Continued

Author	*Date*	*Criterion*	*Variable*	*N*	*Place*	*Correlation*
Ross	1925	Marks	Grade School Composite	134	New Rochelle, N.Y. H.S.	.42–.51
Ross	1925	Marks	Grade School Composite	79	Des Moines, Ia. H.S.	.51–.55
Ross	1925	Marks	Arith. Marks in Various Grades	134	New Rochelle, N.Y. H.S.	.10–.39
Ross	1925	Marks	English Marks in Various Grades	134	New Rochelle, N.Y. H.S.	.09–.34
Ross	1925	Marks	Elementary School Composite	134	New Rochelle, N.Y. H.S.	.42–.51
Lee & Hughes	1934	Pol. Res. Bur. Alg. Test	Hughes Trait Rating Scale	197		.39
Lee & Hughes	1934	Pol. Res. Bur. Alg. Test	Chronological Age	197		.35
Lee & Hughes	1934	Teachers' Marks	Hughes Trait Rating Scale	197		.60
Lee & Hughes	1934	Teachers' Marks	Chronological Age	197		.39
Ross	1925	Marks	Attendance, grades 7 and 8	134	New Rochelle, N.Y. H.S.	.01
Ross	1925	Marks	Deportment, Grades 7 and 8	134	New Rochelle, N.Y. H.S.	.14
			COMBINATIONS			
Dickter	1933	Breslich Test	I.Q. (Otis) and Roger's Test of Math. Ability	83	Jr. H.S., Upper Darby, Pa	.66
Dickter	1933	Breslich Test	I.Q. (Otis) and 8th Grade Arith. Marks	83	Jr. H.S., Upper Darby, Pa.	.70
Dickter	1933	Breslich Test	Roger's Test and 8th Grade Marks	83	Jr. H.S., Upper Darby, Pa.	.73
Dickter	1933	Breslich Test	Roger's Test, 8th Grade Marks and I.Q. (Otis)	83	Jr. H.S., Upper Darby, Pa.	.74
French	1927	Marks	Eighth grade marks, Otis Intelligence Test Score, Stanford Achieve. Test Scores and Age		Martinsville, Ind. H.S.	.49
Glover	1932	Columbia Research Test	I.Q. (Terman) and Orleans Prognostic Test	136	Two Oakland Calif. Jr. High Schools	.65
Kelley	1914	Mark	Alg. Aptitude and Math. Interest Test	123	Two N.Y. City High Schools	.49
Kelley	1914	Mark	4th, 5th 6th and 7th Grade Arith. Marks	59	Two N.Y. City High Schools	.58

Kelley	1914	Mark	Elementary School Grades, Teachers' Character Estimates, Algebra Interest and Aptitude Tests	33	Two N.Y. City High Schools	.81
Kertes	1932	1st Sem. Marks	I.Q. (Terman) and 8th grade Arith. Marks	114	Perth Amboy, N.J.	.66
Kertes	1932	1st Sem. Marks	I.Q. (Terman) and Orleans Prognostic Test	114	Perth Amboy, N.J.	.68
Kertes	1932	1st Sem. Marks	Orleans Prognostic Test and 8th Grade Arith. Marks	114	Perth Amboy, N.J.	.72
Kertes	1932	1st Sem. Marks	I.Q. (Terman) and Orleans Prognostic Test and 8th Grade Arith. Marks	114	Perth Amboy, N.J.	.68
Lee & Hughes	1934	Col. Res. Alg. Test	Lee Aptitude Test for Alg. and Hughes Trait Ratings	197	Three Calif. Jr. High Schools	.66
Lee & Hughes	1934	Col. Res. Alg. Test	Lee Aptitude Test for Alg. and I.Q.—Kuhlman-Anderson	197	Three Calif. Jr. High Schools	.65
Lee & Hughes	1934	Col. Res. Alg. Test	Hughes Trait Ratings and Kuhlman-Anderson I.Q.	197	Three Calif. Jr. High Schools	.59

TABLE II

Coefficients of Correlation between Success in First Year Geometry and Intelligence, Previous Scholastic Achievement, Aptitude Test Scores, and Combinations

Author	*Date*	*Criterion*	*Variable*	*N*	*Place*	*Correlation*
			INTELLIGENCE			
Lee & Hughes	1934	Orleans Geom. Ach. Test	I.Q. (Kuhlman-Anderson)	197	Three Calif. High Schools	.54
Lee & Hughes	1934	Orleans Geom. Ach. Test	I.Q. (Terman)	197	Three Calif. High Schools	.44
Lee & Hughes	1934	Teachers' Marks	I.Q. (Kuhlman-Anderson)	197	Three Calif. High Schools	.31
Lee & Hughes	1934	Teachers' Marks	I.Q. (Terman)	197	Three Calif. High Schools	.26
Orleans (From summary by Brooks)	1928–30	Teachers' Marks	I.Q. (Otis, S.A.)	29–117	Large N.Y. City S.H.	.51 and .54
Cooke-Fields	1932	Marks	Intelligence Test	108		.51
Cooke-Fields	1932	Col. Res. Geom. Test (1st Sem.)	Detroit Adv. Intell. Test	26	Sue Bennet H.S., London, Ky.	.53
Cooke-Fields	1932	Col. Res. Geom. Test (2nd Sem.)	Detroit Adv. Intell. Test	26	Sue Bennet H.S., London, Ky.	.54
Cooke-Pearson	1933	Col. Res. Geom. Test	Terman Intelligence Test	53	Four Missouri High Schools	.67
Cooke-Pearson	1933	Marks	Terman Intelligence Score	53	Four Missouri High Schools	.29
Cooke-Pearson	1933	Col. Res. Geom. Test	Terman Intelligence Score	142	Six Missouri High Schools	.40
Cooke-Pearson	1933	Marks	Terman Intelligence Score	142	Six Missouri High Schools	.48
Dickinson	1925	Marks	Pressey Mental Survey Scale	91	Logansport, Ind. H.S.	.43
			PROGNOSTIC TEST			
Anderson	1929	Marks	Prognostic Test	310	Indianapolis, Ind. H.S.	.67
Cooke-Pearson	1933	Col. Res. Geom. Test	Orleans Prognostic Test	53	Four Missouri High Schools	.70
Cooke-Pearson	1933	Marks	Orleans Prognostic Test	53	Four Missouri High Schools	.22
Cooke-Pearson	1933	Col. Res. Geom. Test	Orleans Prognostic Test	142	Six Missouri High Schools	.43
Cooke-Pearson	1933	Marks	Orleans Prognostic Test	142	Six Missouri High Schools	.52
Dickinson	1925	Marks	Thorndike Visual Voc. Scale	91	Logansport, Ind. H.S.	.41
Dickinson	1925	Marks	Thorndike-McCall Silent Reading Scale	91	Logansport, Ind. H.S.	.47
Lee & Hughes	1934	Orleans Geom. Ach. Test	Lee Test of Geom. Aptitude	197	Three Calif. High Schools	.63
Lee & Hughes	1934	Orleans Geom. Ach. Test	Teachers' Est. of Math. Abil.	197	Three Calif. High Schools	.34

Lee & Hughes	1934	Teachers' Marks	Lee Test of Geom. Aptitude	197	Three Calif. High Schools	.31
Lee & Hughes	1934	Teachers' Marks	Teachers' Est. of Math. Abil.	197	Three Calif. High Schools	.42
Kelly	1914	Marks	Geometry Aptitude Test	85	Two N.Y. City High Schools	.42
Kelly	1914	Marks	Math. Interest Tests	85	Two N.Y. City High Schools	.30
Orleans	1928–30	Col. Res. Bur. Test	Orleans Prognostic Test	29–117	Large N.Y. City High Schools	23 r's 42 to 76 Ave.=58
Orleans	1928–30	Teachers' Marks	Orleans Prognostic Test	29–117	Large N.Y. City High Schools	16 r's 42 to 78 Ave.=62
			TEACHERS' MARKS			
(From summary by Brooks)		Marks	Algebra I Marks	2143		.38 to .70 Ave.=.54
(From summary by Brooks)		Marks	English I Marks	1773		.25 to .67 Ave.=.43
(From summary by Brooks)		Marks	History I Marks	33		.64
(From summary by Brooks)		Marks	French	128		.40 to .53 Ave.=.48
(From summary by Brooks)		Marks	General Science Marks	33		.61
Brown	1909	Marks	Algebra Mark			.66
Burris	1903	Marks	Algebra Mark		16 Schools	.45
Cooke-Fields	1932	Col. Res. Geom. Test 1st Sem.	Stanford Arith. Ach. Test	26	Sue Bennet H.S., London, Ky.	.30
Cooke-Fields	1932	Col. Res. Geom. Test 2nd Sem.	Stanford Arith. Ach. Test	26	Sue Bennet H.S., London, Ky.	.39
Cooke-Pearson	1933	Col. Res. Geom. Test	Algebra Marks	53	Four Missouri High Schools	.09
Cooke-Pearson	1933	Marks	Algebra Marks	53	Four Missouri High Schools	.39
Cooke-Pearson	1933	Col. Res. Geom. Test	Algebra Marks	142	Six Missouri High Schools	.53
Cooke-Pearson	1933	Marks	Algebra Marks	142	Six Missouri High Schools	.55
Crathorne	1922	Marks	Algebra Marks	1900		.52
Rogers	1918	Marks	Algebra Marks			.51
Weglein	1917	Marks	Algebra Marks			.63
Winegarden	1929	Marks	Algebra Marks			.51
Lee and Lee	1931	Rev. from Geom. Test	Non-Standard Algebra Test		Glendale, Calif, H.S.	.47

TABLE II—Continued

Author	*Date*	*Criterion*	*Variable*	*N*	*Place*	*Correlation*
Lee and Lee	1931	Rev. from Geom. Test	Non-Standard Algebra Test		Burbank, Calif. H.S.	.71
Lee and Lee	1931	Marks	Marks		Glendale, Calif. H.S.	.41
Lee and Lee	1931	Marks	Marks		Burbank, Calif. H.S.	.57
Lee & Hughes	1934	Orleans Geom. Ach. Test	Hughes Trait Rating Scale	197	Three Calif. Jr. High Schools	.37
Lee & Hughes	1934	Teachers' Marks	Hughes Trait Rating Scale	197	Three Calif. Jr. High Schools	.35
			COMBINATIONS			
Cooke-Pearson	1933	Col. Res. Geom. Test	Orleans Prognostic Test and Terman Intelligence Score	53	Four Missouri High Schools	.73
Cooke-Pearson	1933	Col. Res. Geom. Test	Orleans Prognostic Test and Algebra Marks	53	Four Missouri High Schools	.70
Cooke-Pearson	1933	Col. Res. Geom. Test	Terman Intelligence Score Algebra Marks	53	Four Missouri High Schools	.68
Cooke-Pearson	1933	Col. Res. Geom. Test	Orleans-Terman and Agl. Marks	53	Four Missouri High Schools	.75
Cooke-Pearson	1933	Final Marks	Orleans and Terman	53	Four Missouri High Schools	.29
Cooke-Pearson	1933	Final Marks	Orleans and Algebra Marks	53	Four Missouri High Schools	.41
Cooke-Pearson	1933	Final Marks	Terman and Alg. Marks	53	Four Missouri High Schools	.44
Cooke-Pearson	1933	Final Marks	Orleans-Terman and Alg. Marks	53	Four Missouri High Schools	.44
Cooke-Pearson	1933	Col. Res. Geom. Test	Orleans and Terman	142	Four Missouri High Schools	.44
Cooke-Pearson	1933	Col. Res. Geom. Test	Orleans and Algebra Marks	142	Four Missouri High Schools	.50
Cooke-Pearson	1933	Col. Res. Geom. Test	Terman and Algebra Marks	142	Four Missouri High Schools	.49
Cooke-Pearson	1933	Col. Res. Geom. Test	Orleans, Terman and Alg. Marks	142	Four Missouri High Schools	.51
Cooke-Pearson	1933	Final Marks	Orleans and Terman	142	Four Missouri High Schools	.53
Cooke-Pearson	1933	Final Marks	Orleans and Algebra Marks	142	Four Missouri High Schools	.62
Cooke-Pearson	1933	Final Marks	Terman and Algebra Marks	142	Four Missouri High Schools	.42
Cooke-Pearson	1933	Final Marks	Orleans, Terman and Alg. Marks	142	Four Missouri High Schools	.63
Kelley	1914	Marks	Geom. Aptitude & Math. Interest	85	Four New York City H. S.	.44
Lee & Hughes		Col. Res. Geom. Test	Lee Geom. Aptitude Test and Hughes Trait Ratings	197	Three Calif. Jr. High Schools	.67
Lee & Hughes		Col. Res. Geom. Test	Lee Geom. Aptitude Test and I.Q. (Kuhlman-Anderson)	197	Three Calif. Jr. High Schools	.66
Lee & Hughes		Col. Res. Geom. Test	Hughes Trait Ratings and I.Q. (Kuhlman-Anderson)	197	Three Calif. Jr. High Schools	.57

TABLE III

Coefficients of Correlation between Success in the Upper Years or all High School Mathematics and Intelligence, Scholastic Achievement in Previous Years and Other Factions

Author	*Date*	*Criterion*	*Variable*	*N*	*Place*	*Correlation*
Flack	1926	Ballard's Standard Tests	Various Memory and Other Mental Tests	28	English Sec. School	Negligible
Flemming	1925	Marks for the Year	Terman Intelligence	121	Horace Mann School	.49
Flemming	1925	Marks for the Year	Miller Intelligence	girls in 3	Horace Mann School	.38
Flemming	1925	Marks for the Year	Otis, S.A. Intelligence	grades	Horace Mann School	.52
Flemming	1925	Marks for the Year	Haggerty Reading 3A	7, 8 and 9	Horace Mann School	
Flemming	1925	Marks for the Year	Haggerty Reading 3A	165	Horace Mann School	.41
Flemming	1925	Marks for the Year	Terman Intelligence	girls in 3	Horace Mann School	.58
Flemming	1925	Marks for the Year	Miller Intelligence	grades	Horace Mann School	.61
Flemming	1925	Marks for the Year	Otis, S.A.	10, 11	Horace Mann School	.53
Flemming	1925	Marks for the Year	Haggerty Reading 3A	and 12	Horace Mann School	.15
Koenig	1928	Marks	Terman Scores		Jr. H.S., Mason Town, Pa.	.33
Koenig	1928	Marks	English Marks		Jr. H.S., Mason Town, Pa.	.57
Shewman	1926	Four Year Math. Marks	I.Q. (Terman)	191	Grover Cleveland H.S. St. Louis	.49
Shewman	1926	Four Year Math. Marks	Arith. Section of Terman	191	Grover Cleveland H.S. St. Louis	.58
Shewman	1926	Four Year Math. Marks	Number Series Section of Terman	191	Grover Cleveland H.S. St. Louis	.30
Unziker	1925	Marks for the Y	Index of Brightness (Otis Intell. Test)	425	Garfield Jr. H.S. Fond du Lac, Wis.	.34
HIGH SCHOOL MATHEMATICS						
Jordan	1922	Marks	Otis Group Intell. Test	67	Univ. of Ark. H.S.	.28
Jordan	1922	Marks	Army Alpha Intell. Test	67	Univ. of Ark. H.S.	.37
Jordan	1922	Marks	Miller Intell. Test	67	Univ. of Ark. H.S.	.34
Jordan	1922	Marks	Terman Intell. Test	67	Univ. of Ark. H.S.	.35
(Reported by Brooks)		Marks	Intelligence Test	315		

TABLE III—Continued

Author	*Date*	*Criterion*	*Variable*	*N*	*Place*	*Correlation*
SECOND YEAR ALGEBRA						
(From summary by Brooks)			Intelligence Test	108		.49
			Algebra I Marks	473		.61 to .74 Ave. 67
			English I Marks	82		.25 to .44 Ave. 35
			Gen. Science Marks	49		.25 .55
			French I Marks	33		.53
			Av. Mark First Year	33		.64
			English II Marks	33		.39
			Geometry Marks	313		.47 to .73 Ave. .61
			Av. Mark Second Year	33		.57
			Age	108		.24
(From summary by Brooks)			Algebra and Geometry Marks	108		.75
			Algebra Marks and Intelligence	108		.68
			Geom. Marks and Intelligence	108		.66
			Alg. and Geom. Marks and Intell.	108		.75
TRIGONOMETRY						
(From summary by Brooks)			Algebra I	41		.69
			Algebra II	41		.70
			Geometry	40		.66
			Age	41		.09

Mathematics and Poultry Farming

By HAROLD A. SMITH '36
State Normal School, Oneonta, New York

JUDGING FROM accurate records, the depression has not seriously affected the poultry farmer. However, it is not my purpose to show the economic condition of the poultry farmer, but to show that he uses mathematics.

The following is a portion of a page from a record book kept by an up-to-date systematic poultry man:

Day	*Number of eggs*	*Number of cases shipped*	*Kinds*	*Price per doz.*	*Gross*	*Feed*
1	615		90l	45½	195.50	93.11
2	667	6	60m	32	68.25	24.50
3	631		30s	27		
4	636		90l	42		
5	655	6	90m	32	66.60	35.35
					330.35	152.96
					152.96	
				Net	297.39	

It will be noticed that several mathematical processes are used. In another case, in a certain month, the total number of cases of eggs was 54. This makes 1620 doz. eggs. The net income for the month was $550.35. Dividing the net income for the month by the number of dozen eggs, we find that the average price per dozen was 34¢.

A flock should lay about 50%. In order to find whether a flock is laying well or not, the following computation is needed:

Original number of hens	80
Died	4
Hens left	76
Eggs produced in a certain day	64

64/76=85%.

My own experience on a poultry farm brought in the following problems:

1. Peat moss costs $6.00 for a bale large enough to cover an area 100 ft. square. This lasts for six months after which it can be sold at a reasonable rate as fertilizer. Straw costs $1.00 a bale and

covers twice as much area. It has to be changed once a month. Thus

Using peat moss		*Using straw*	
Cost of bale	$6.00	Cost of three bales	$3.00
Labor	1.00	Labor	6.00
Total cost	7.00	Total	9.00
Sale value	1.50	Sale value	3.00
Net cost	5.50	Net cost	6.00

If a farmer considered his labor, it is cheaper to use peat moss.

2. Statistics show that broilers are worth about 10¢ per lb. in October. Since they should be ready for market in four months, they should be started in June or July. It was also shown by statistics that broilers ready by the Fourth of July were worth about twice as much. This meant that they would have to be started as early as March. In order to find which was better, a great deal of mathematics was used. "Ole man weather" had to be accounted with in figuring the possible cost of coal necessary for extra heat that would be required. The initial cost of early chicks is higher and the probable death rate greater.

3. I doubt if anyone on a poultry farm has found anything better to combat lice than black leaf "40." A certain amount of this will cover a certain length of roost. Before using this, it was handy to find how large a quantity to buy. The estimate was made by mathematics.

4. One of the most important phases of the poultry business is to market the products. We found that we could get 4 or 5 cents per dozen more by taking the eggs to Schenectady than by shipping them to New York. We then had to figure the cost of crates, the shipping rate, the amount of gasoline consumed, and the other expenses of a car and last and also least the amount of the labor required.

On to St. Louis

All members of the National Council of Teachers of Mathematics should plan to attend the St. Louis meeting of the Council. The program follows on page 507.

PROGRAM OF THE SEVENTEENTH ANNUAL MEETING OF THE NATIONAL COUNCIL OF TEACHERS OF MATHEMATICS, ST. LOUIS, MISSOURI

December 31, 1935–January 1, 1936

Tuesday Morning Session, December 31, 9:00 A.M.; Room 320 Commerce and Finance Building, St. Louis University.

Joint Session with the Mathematical Association of America and Section A of the American Association for the Advancement of Science.

1. Report of Joint Commission on the Place of Mathematics in the Secondary Schools. Professor Kenneth P. Williams, Indiana University, Chairman.

2. Open discussion.

3. The Main Purposes and Objectives in Teaching High School Mathematics. Mr. William Betz, Rochester, New York, representing the National Council of Teachers of Mathematics. Professor W. W. Hart, University of Wisconsin, representing the Mathematical Association of America.

4. Open discussion.

Meeting of Board of Directors, December 31, 11:30 A.M. Parlor D, Coronado Hotel followed by Board of Directors Luncheon, 12:30 P.M., Pine Room, Coronado Hotel.

Tuesday Afternoon Meeting, December 31, 2:00 P.M.; Crystal Room, Coronado Hotel.

SYMPOSIUM ON THE TEACHING OF GEOMETRY

1. The Purpose, Nature, and Use of Pictures in the Teaching of Solid Geometry. Professor W. H. Roever, Washington University, St. Louis, Missouri.

2. Stereoscopy as an Aid to the Teaching of Solid Geometry. Mr. John T. Rule, the Taylor School, Clayton, Missouri.

3. Developing the Meaning of Demonstration in Geometry. Mr. Rolland R. Smith, Springfield, Massachusetts.

Tuesday Evening Session, December 31, 8:00 P.M.: Crystal Room, Coronado Hotel.

PROGRAM SESSION

1. Address of Welcome. President Robert S. Johnson of St. Louis University.

2. Response for the Council: Miss Edith Woolsey, Member of the Board of Directors, Minneapolis.

3. The History of the Development of the Metric System. (Illustrated.) Professor Edwin W. Schreiber, State Teachers College, Macomb, Illinois.

4. Mathematical Recreations, an Aid or a Relief? Miss Ruth Lane, University High School, Iowa City, Iowa.

Wednesday Morning Session, January 1, 1935, 9:00 A.M.; Crystal Room, Coronado Hotel.

9:00–10:30 A.M. Annual Business Meeting

1. Report of Officers and Committees.
2. Election of Officers.
3. A Communication from the Amherst Conference of Private School Teachers of Mathematics, presented by Mr. Rolland R. Smith.

10:45–11:50 A.M. Program Session

4. Functional Thinking and Teaching in Secondary School Mathematics. Professor H. C. Christofferson, Miami University, Oxford, Ohio.
5. The Crisis in Mathematics—at Home and Abroad. W. D. Reeve, Editor of *The Mathematics Teacher*.

Wednesday Afternoon Session. 12:30–2:30 P.M. Parlor D, Coronado Hotel.

Luncheon Meeting of Official Delegates* and Board of Directors including a round table discussion of Council policies.

Wednesday Evening Meeting, January 1, 7:00 P.M.; Crystal Room, Coronado Hotel.

ANNUAL BANQUET

Babylonian Mathematics, with Special Reference to Recent Discoveries. Professor Raymond Claire Archibald, Brown University.

WEDNESDAY AFTERNOON AFTER 2:30 P.M. IS RESERVED FOR VISITS TO POINTS OF INTEREST IN ST. LOUIS UNDER GUIDANCE OF THE LOCAL COMMITTEE

The Council is invited to participate in the session of Section Q (Education) of the A.A.A.S. on Monday Morning Dec. 30, at 9:30 at the Statler Hotel. A portion of the program has to do with the teaching of arithmetic. At this meeting Professor Raleigh Schorling past President of the National Council will speak on "Needed Research on the Problem of the Slow Learning Pupil."

* This meeting is open to all members of the Council who wish to attend. Make your reservations early. Those who do not care to attend the luncheon are especially invited to join the group at the close of the luncheon period (1:30 P.M.).

NEWS NOTES

THE National Council of Teachers of Mathematics held its First Summer Meeting at Denver, Colorado from June 29 to July 1, 1935.

PROGRAM

Saturday, June 29—2:00 P.M.

Argonaut Hotel
General Meeting—Vice-president A. R. Congdon Presiding

1. Welcome to Colorado. H. W. Charlesworth, East High School, Denver, Colorado.
2. The Trend Toward Fusion Mathematics as Shown by an Analysis of Junior High School Texts. Carl S. Thomas, Nebraska State Normal College, Chadron, Nebraska.
3. Weaknesses and Suggested Remedies in Objective Tests. Martha T. Denny, Classen High School, Oklahoma City, Oklahama.
4. A High School Teacher's View on the training of Teachers of Secondary Mathematics. Martha Hildebrandt, Proviso Township High School, Maywood, Illinois.
5. Geometric Design (Illustrated). A. Bruce Ewer, Garfield School, Johnstown, Colorado.

Saturday, June 29—6:30 P.M.

Argonaut Hotel
Banquet
Dinner Music—Art Trio, Denver, Colorado

1. Announcements.
2. Violin Selections. George Wagner, East High School, Denver, Colorado.
3. Address: Crises in Education, Economics, and Mathematics. Dr. Earle R. Hedrick, University of California, Los Angeles, California.

Sunday, June 30—9:45 A.M.

Echo Lake Outing—Mountain Trip and Picnic

Monday, July 1—8:00 A.M.

Argonaut Hotel
Breakfast

1. Announcements.
2. The Human Side of Mathematics. Mary A. Potter, Supervisor of Mathematics, Racine, Wisconsin.
3. Round Table Discussion.

Monday, July 1—2:00 P.M.

Auditorium, East High School
General Meeting—Department of Secondary Education
Theme: High School Education and Youth Problems of Today

Monday, July 1—3:00 P.M.

Room 304, East High School
Joint Conference—Department of Secondary Education in Cooperation with the National Council
Vice-President A. R. Congdon Presiding
Central Topic—How Can Mathematics Contribute to the Solution of Present Youth Problems?

1. L. W. Lavengood, Public Schools, Tulsa, Oklahoma.
2. G. W. Finley, State College of Education, Greeley, Colorado.
3. Discussion. Leader: J. C. Stearns, University of Denver, Denver, Colorado.

THE FOLLOWING letter which was sent out recently should be of interest to all mathematics teachers:

THE UNIVERSITY OF WISCONSIN
Madison

Department of Mathematics

To Teachers of Mathematics
in Wisconsin High Schools.

Dear Colleagues:

There have been so many rumors concerning changes in the entrance requirements to the University of Wisconsin and especially that portion of the requirements which relates to mathematics that I am writing to you to set forth the facts.

Heretofore the requirements in mathematics for entrance to the University have been one unit of algebra and one unit of geometry with the single exception that students of engineering must have had an additional half unit of algebra for unconditional entrance to the College of Engineering.

The University has now established two classes of entrance, "unrestricted" and "restricted." For "unrestricted" entrance the requirements in mathematics have been left unchanged. A student securing "restricted" admission can major in only a limited number of subjects. These are essentially literature and language, history, journalism and a few special courses such as art history, art education, and music. Such a student cannot take any mathematics in residence in the University, and it is probable that he will be unable to register in any courses in physics or chemistry. He cannot major in any science, in sociology, in economics, in political science, in philosophy, or enter the College of Engineering or the College of Agriculture. If he enters the School of Education he will be debarred from even minoring in any of these subjects.

In addition, Mr. Callahan, Superintendent of Schools, has ruled that, since for the vast majority of students mathematics is in effect to be an entrance requirement, the high schools must continue to offer algebra and geometry.

Of course a student may change his status from "restricted" to "unrestricted" by work in correspondence or private tutoring followed by examinations.

If the mathematicians of the State are alert, the situation affords us a splendid opportunity to emphasize the importance and value of mathematics. Without any solicitation the great majority of the various departments of the University said that a mathematical training was a necessity for their work; and many other departments, such as Comparative Literature, Latin, and the History of Art expressed the desire that their students know mathematics. A student should be made to realize that his freedom of activity in the University as well as in life will be greatly hampered if he neglects to study the fundamental portions of mathematics in the high school.

The Department of Mathematics hopes that you bring the force of the distinction between "restricted" and "unrestricted" entrance to the attention of your high school principal and other school authorities. They will no doubt already be in possession of the new entrance regulations which, however, contain much material not related to mathematics.

If we make immediate use of the voluntary recognition which we have received from those who understand the importance of our subject in modern intellectual activity, we can turn the current attacks on mathematics back upon themselves and gain rather than lose by the new entrance requirements.

Sincerely

MARK H. INGRAHAM, *Chairman*

THE TWENTY-FIRST annual meeting of the Kansas Section of the Mathematics Association of America and the thirty-first annual meeting of the Kansas Association of Mathematics Teachers was held at the Topeka High School on March 16, 1935. The program follows:

Forenoon Session

Joint Session—K.A.M.T. and M.A.A.

Chairman, Guy W. Smith,
University of Kansas

1. Probability Distributions. J. A. G. Shirk, Kansas State Teachers College, Pittsburg.
2. Correlation of Junior and Senior High School Mathematics. Edward Kaufman, Kingman.
3. The Status of Students Entering College Without High School Mathematics. H. E. Jordan, University of Kansas, Lawrence.
4. The Trigonograph. R. D. Daugherty, Kansas State College, Manhattan.

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Afternoon Session

Joint Session—K.A.M.T. and M.A.A.

1. Business Session
   Report of Secretary-Treasurer. Dessie Myers, Kansas City.
   Report of the Nominating Committee and Election of Officers. Edna Austin, Topeka.

Afternoon Session

K.A.M.T.

Chairman, Delbert Emery
Manhattan

Topic: The Need for Re-Orientation of Mathematics in the Secondary Schools.

1. From the Viewpoint of Modern Educational Theory. W. H. Hill, Kansas State Teachers College, Pittsburg.
2. From the Viewpoint of the University Professor of Mathematics. E. W. Ploenges, Kansas Wesleyan University, Salina.
3. From the Viewpoint of the High School Teacher. J. T. LaRue, Junction City.

Afternoon Session

M.A.A.

Chairman, Guy W. Smith,
University of Kansas, Lawrence

1. The Stationary Singularities of Rational Curves. Oscar J. Peterson, Kansas State Teachers College, Emporia.
2. Systems of Linkages. W. C. Janes, Kansas State College, Manhattan.
3. Business Session.
   Report of the Nominating Committee and Election of Officers.
4. Calculation of Circulating Decimals. U. G. Mitchell, University of Kansas, Lawrence.

THE FOLLOWING review of the tenth yearbook of the National Council of Teachers of Mathematics on "The Teaching of Arithmetic" will be of interest to the readers of the *Mathematics Teacher:*

The general character of these books which the National Council of Teachers began to publish in 1926 was described in detail in a notice in the *Gazette*, December 1932, and the current volume is compiled on the same lines. They differ from the Reports of the Mathematical Association in two important respects: (i) the various chapters in the book are contributed by individual teachers and are not designed to represent collective opinion; (ii) there are numerous detailed statistical accounts of experiments which have been carried out to compare the efficiency of various methods.

This volume is concerned mainly with the first stages of arithmetic, and a large

part of it is devoted to the consideration of the value of the drill element: the general conclusion that elementary arithmetic should be regarded less as a challenge to the pupil's memory and more as a challenge to his intelligence indicates briefly the character of the discussion. There is an interesting chapter on teacher-training courses which shows the great variety of methods employed; and it is evident that at present there is no general agreement among those responsible for such courses as to what system is most effective.

These yearbooks certainly deserve a place in every reference library for teachers.

C. V. D.

THE Mathematics Section of the Central Ohio Teachers Association met in Dayton, Ohio on Friday, October 25, 1935 at 2 P.M. The program follows:

Mr. Thurl Stephens, Piqua, Chairman
Miss Myrtle Weldasin, Springfield, Vice-Chairman
Miss Pauline Sperrow, Mt. Vernon, Secretary
Address—"Mathematics in Life." J. C. Boldt, Steele High School, Dayton.
Address—"Future Trends in Teaching Mathematics." Dr. W. D. Reeve, Teachers College, Columbia University.
Election of Officers

The Fall meeting of the Connecticut Valley Section of the Association of Teacher of Mathematics in New England was held at Northfield Seminary in East Northfield, Massachusetts on Saturday October 5, 1935.

PROGRAM

Morning Session

10:00—Social Gathering.
10:15—Welcome. Miss Mira B. Wilson, Principal of Northfield Seminary.
"Mathematics in English Schools." Miss Harriet Howard, Northfield Seminary.
"Geometry in the new Alpha and Beta Examinations." Mr. Ray F. Farnsworth, Chauncy Hall School, President of the Association of Teachers of Mathematics in New England.
"Report of The Amherst Conference on Liberalizing the Teaching of Secondary Mathematics." Miss Helen Wright.
12:00—Luncheon.

Afternoon Session

1:15—Business Meeting.
"Possible, but - - - ." Professor William R. Ransom, Tufts College.
"New Tendencies in Secondary Mathematics." Professor W. D. Reeve, Columbia University.

Officers of the Connecticut Valley Section

Nelson A. Jackson, President, Mount Hermon School.
Fred L. Mockler, Vice-President, High School, Holyoke.
Miss Helen Wright, Secretary, Northfield Seminary.
Miss Beatrice Neale, Treasurer, Bulkeley High School, Hartford.
Professor David D. Leib, Director, Connecticut College.
Miss Sarah K. Evarts, Director, Bulkeley High School, Hartford.

THE FOLLOWING note appeared on the above program:

"Every teacher of mathematics should be a member of the National Council of Teachers of Mathematics and a regular subscriber to the *Mathematics Teacher*."

It would be a good plan for other groups to adopt this idea in printing their programs so as to calll attention to this important matter.

THE MATHEMATICS SECTION of The Maryland State Teachers Association

held its Spring Meeting at Hagerstown on April 27, 1935. The program follows:

Officers

Chairman—Mr. Buck, Principal, Kenwood High School, Raspeburg P. O.
Secretary—Miss Hester Whitfield, Forest Park High School, Baltimore, Md.
Treasurer—Miss Agnes Herbert, Clifton Park Junior High School, Baltimore, Md.

Afternoon Program

1. Procedures with dull-normal groups. Miss Anna Campbell, Hamilton, J. H. S., Baltimore City.
   Procedures with Gifted Pupils. Mr. Robert Wilson, Principal Bladensburg J. H. S., Prince George Co.
2. Shall instruction in mathematics be primarily directed toward preparation for advanced studies, or should it be primarily concerned with the value of its own courses, regardless of the pupils' future academic career? Mr. Arthur Taylor, Alleghany H. S., Cumberland, Alleghany Co.
3. In order to make learning in mathematics functional, what obsolete material should be omitted? Miss Margaret Hamilton, Penna. Ave. H. S., Cumberland, Alleghany Co.
   What mathematics should be required and what made elective? Mr. Weldon Dawson, Principal Clarksville H. S., Howard Co.
4. Mathematics and the integrated program. Mr. Douglas Bivens, Principal Hancock H. S., Washington Co.
5. The recreational aspects of mathematics. J. H. S., Miss Anna Meeks, Franklin H. S., Baltimore Co. S. H. S., Miss Alverta Dillon, Oakland H. S., Garrett Co.
6. Higher standards for teachers. Mr. W. J. Stenger, Chestertown H. S., Kent Co.
7. The limitations imposed by small enrollment upon revision of the mathematics curriculum. Mr. Bayard Ayres, Indian Head H. S., Charles Co.
8. Summary. Mr. William Pyle, Jarrettsville, Hartford Co.

Banquet—6 P.M.
Alexander Hotel

1. Announcements.
2. Address—Dr. Wm. David Reeve, Teachers College, Columbia University.

THE Association of Mathematics Teachers of New Jersey held its Fifty-fourth Regular Meeting at The State University of New Jersey in New Brunswick on May 4, 1935.

Officers of the Society

Dr. Frank J. McMackin, President, Dickinson High School, Jersey City.
Professor Albert E. Meder, Vice-President, New Jersey College for Women, New Brunswick.
Andrew S. Hegeman, Secretary-Treasurer, Central High School, Newark.

Program

Title: A vitalized mathematics for the 9th grade; being a discussion of a course of study in mathematics for the 9th grade as developed by the State Committee on Mathematics. Teachers who have been experimenting with the course will comment on the results they have obtained with it.
Objectives and Philosophy (8 Minutes). Mr. William W. Strader, Public School No. 12, Jersey City, N. J.
Direct Measurement (8 Minutes). Mr. Roscoe P. Conkling, Central High School, Newark, N. J.
Formula (8 Minutes). Miss Amanda Loughren, Thomas Jefferson High School, Elizabeth, N. J.
Graphs (8 Minutes). Mr. David E. Amidon, West Side High School, Newark, N. J.

Opposite Quantities (8 Minutes). Mr. Howard Hart, East Orange High School, East Orange, N. J.

Mathematics of the Home (8 Minutes). Dr. Frank J. McMackin, Dickinson High School, Jersey City, N. J. Reactions of teachers who have been trying the course in their classes.

Mr. Morris Savage (3 Minutes). Central High School, Paterson, N. J.

Miss Frances Soman (3 Minutes). Dickinson High School, Jersey City, N. J.

Miss Mary Rogers (3 Minutes). Roosevelt Jr. High School, Westfield, N. J.

Mr. John Dwyer (3 Minutes). Washington Junior High School, Elizabeth, N. J.

Miss Catherine Read (3 Minutes). Junior School No. 1, Trenton, N. J.

Mr. George M. Ramage (3 Minutes). Linden High School, Linden, N. J.

Report of the Council

Election of Officers

THE ANNUAL Spring meeting of the Mathematics Section of the California Teachers Association, Bay Section, was held on Saturday, May 18, 1935, at 10:00 A.M., in San Francisco.

Dr. Edwin A. Lee, Superintendent of School, San Francisco, spoke on "Mathematics in a Modern Curriculum." A question and discussion period followed Dr. Lee's address.

A short business meeting was held at which the following were elected to serve as officers of the Section for the year 1935–36:

Chairman: Miss Helen T. Hoefer, Richmond Union High School, Richmond, California.

Vice Chairman: Mrs. Lulu V. Schott, Mission High School, San Francisco.

Secretary-Treasurer: Miss Ruth Peterson, Richmond Union High School, Richmond, California.

THIS YEAR has seen a great increase in attendance at the meetings of the Men's Mathematics Club of Chicago and the Metropolitan Area, indicating a healthy growth of interest in the subject of the teaching of mathematics.

Three speakers took part in the program for the April meeting. Mr. C. M. Austin of the Oak Park High School gave a brief history of the Chicago Men's Mathematics Club. Mr. J. T. Johnson of the Chicago Normal College gave a short report of the National Council meetings at Atlantic City. Dr. W. C. Krathwohl of the Armour Institute of Technology spoke on "Mathematics used in the designing of lenses in optical systems." It is interesting to note that this application in which only the simplest formulas are used offers a splendid correlation between physics and mathematics.

WILLIAM C. KRATHWOHL
Secretary

The Coronado Hotel, the headquarters for the meetings of the National Council of Mathematics Teachers, is located at the corner of Lindell Blvd. and Spring Ave. Those who arrive at the Union Station should plan to take the 18th Street carline north to Olive St. There they should transfer to either the Maryland, University, or Delmar Olive cars going west. They should get off the car at Spring Ave. and Olive St. which is about 3700 west. They should then walk one short block south to the hotel which is on the northwest corner of Spring and Lindell Blvd. Those who wish to use the bus, should take the 18th Street car to Locust St. There they will be able to get the Lindell Bus going west which will take them directly out Lindell Blvd. to the hotel.

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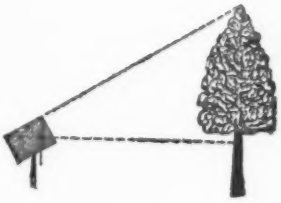

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Reprints

Art and Mathematics, by *Professor Nathan Altshiller Court*—Price 20¢

A Glance at Some of the Ideas of Charles Sanders Peirce, by *Professor Cassius Jackson Keyser*—Price 35¢

Mathematics and the Dance of Life, by *Professor Cassius Jackson Keyser* Price 20¢

The Meaning of Mathematics, by *Professor Cassius Jackson Keyser*—Price 15¢

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1. *The Mathematics Teacher.* Published monthly *except in June, July, August and September.* It is the only magazine in America dealing exclusively with the teaching of mathematics in elementary and secondary schools. Membership (for $2) entitles one to receive the magazine free.

2. *The National Council Yearbooks.* The first Yearbook on "A General Survey of Progress, in the last Twenty-five Years" is out of print. The second on "Curriculum Problems in Teaching Mathematics" may be secured for $1.25. The third on "Selected Topics in Teaching Mathematics," the fourth on "Significant Changes and Trends in the Teaching of Mathematics Throughout the World Since 1910," the fifth on "The Teaching of Geometry," the sixth on "Mathematics in Modern Life," the seventh on "The Teaching of Algebra," the eighth on "The Teaching of Mathematics in Secondary Schools," the ninth on "Relational and Functional Thinking in Mathematics," and the tenth on "The Teaching of Arithmetic"—each may be obtained for $1.75 (bound volumes), from the Bureau of Publications, Teachers College, 525 West 120th Street, New York City.

II. The Editorial Committee of the above publications is W. D. Reeve of Teachers College, Columbia University, New York, Editor-in-Chief; Dr. Vera Sanford, of the State Normal School, Oneonta, N.Y.; and H. E. Slaught of the University of Chicago.

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Alice in Dozenland. Pitcher, Wilhelmina, XXVII, Dec. 1934.

A Near Tragedy. Miller, Florence Brooks, XXII, Dec. 1929.

An Idea That Paid. Miller, Florence Brooks, XXV, Dec. 1932.

If. Snyder, Ruth L. XXII, Dec. 1929.

Mathematical Nightmare. Skerrett, Josephine, XXII, Nov. 1929.

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The Eternal Triangle. Raftery, Gerald, XXVI, Feb. 1933.

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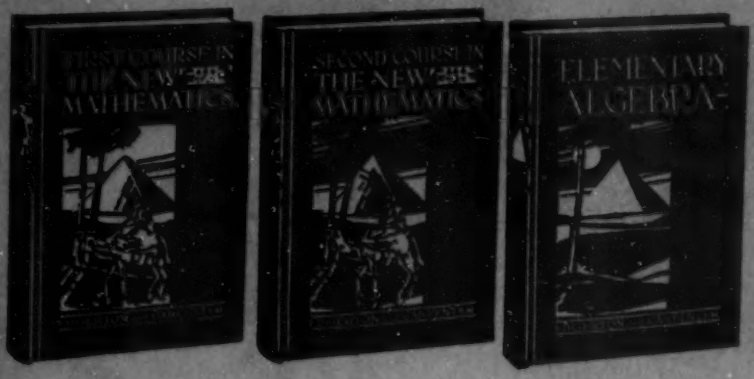

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